Then Fourier sine transform of I^{st} derivative $\frac{df}{dt}$ is

$$g_{IS}(\omega) = \sqrt{\frac{2}{\pi}}\int_0^\infty \frac{df}{dt}\sin\omega t\,dt$$

$$= \sqrt{\frac{2}{\pi}}\left[f(t)\sin\omega t\right]_0^\infty - \sqrt{\frac{2}{\pi}}\cdot\omega\int_0^\infty f(t)\cos\omega t\,dt$$

As I^{st} term vanishes since $f(t)\to 0$ as $t\to\infty$ and using (2), above eqn becomes

$$g_{IS}(\omega) = -\omega g_c(\omega) \quad -(3)$$

where $g_c(\omega)$ is fourier cosine transform of $f(t)$.

Also the Fourier cosine transform of I^{st} derivative of $f(t)$ is

$$g_{Ic}(\omega) = \sqrt{\frac{2}{\pi}}\int_0^\infty \frac{df}{dt}\cos\omega t\,dt$$

$$= \sqrt{\frac{2}{\pi}}\left[f(t)\cos\omega t\right]_0^\infty + \sqrt{\frac{2}{\pi}}\omega\int_0^\infty f(t)\sin\omega t\,dt$$

$$= -\sqrt{\frac{2}{\pi}}f(0) + \omega g_s(\omega) = \omega g_s(\omega) - \sqrt{\frac{2}{\pi}}f(0) \quad -(4)$$

where $g_s(\omega)$ is the Fourier sine transform of $f(t)$.

Again, Fourier sine transform of 2^{nd} derivative of $f(t)$ ie $\frac{d^2f}{dt^2}$ is given by

$$g_{2s}(\omega) = \sqrt{\frac{2}{\pi}}\int_0^\infty \frac{d^2f}{dt^2}\sin\omega t\,dt$$

$$= \sqrt{\frac{2}{\pi}}\left[\frac{df}{dt}\sin\omega t\right]_0^\infty - \sqrt{\frac{2}{\pi}}\cdot\omega\int_0^\infty \frac{df}{dt}\cos\omega t\,dt$$

As first term vanishes since $f(t)\to 0$ as $t\to\infty$, we get

$$g_{2s}(\omega) = -\omega g_{1c}(\omega)$$

$$= -\omega\left[\omega g_s(\omega) - \sqrt{\frac{2}{\pi}}f(0)\right]$$

$$= -\omega^2 g_s(\omega) + \sqrt{\frac{2}{\pi}}\omega f(0) \quad -(5)$$

Similarly, Fourier cosine transform of 2^{nd} derivative of $f(t)$ is

$$g_{2c}(\omega) = \sqrt{\frac{2}{\pi}}\int_0^\infty \frac{d^2f}{dt^2}\cos\omega t\,dt = \sqrt{\frac{2}{\pi}}\left[\frac{df}{dt}\cos\omega t\right] + \sqrt{\frac{2}{\pi}}\omega\int_0^\infty \frac{df}{dt}\sin\omega t\,dt$$

$$= -\sqrt{\frac{2}{\pi}}\, f'(0) + \omega g_{1s}(\omega)$$

$$= -\sqrt{\frac{2}{\pi}}\, f'(0) - \omega^2 g_c(\omega) \quad \text{— (6)}$$

(using 3.

Example 1 :- Find the Fourier transform of the slit function $f(x)$ defined as

$$f(x) = \begin{cases} 1/\epsilon & |x| \le \epsilon \\ 0 & |x| > \epsilon \end{cases}$$

Determine the limit of this transform as $\epsilon \to 0$ and discuss the result.

Solution The fourier transform of function $f(x)$ is

$$g(\omega) = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{+\infty} f(x)\, e^{-i\omega x}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\epsilon}^{+\epsilon} \frac{1}{\epsilon} e^{-i\omega x} dx = \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\epsilon} \left[\frac{e^{-i\omega x}}{-i\omega} \right]_{-\epsilon}^{+\epsilon}$$

$$= \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\epsilon} \, \frac{e^{i\omega\epsilon} - e^{-i\omega\epsilon}}{i\omega} = \frac{1}{\sqrt{2\pi}} \, \frac{2}{\omega\epsilon} \, \frac{e^{i\omega\epsilon} - e^{i\omega\epsilon}}{2i}$$

$$= \sqrt{\frac{2}{\pi}} \, \frac{\sin \omega\epsilon}{\omega\epsilon}$$

This is required Fourier transform of given fn.

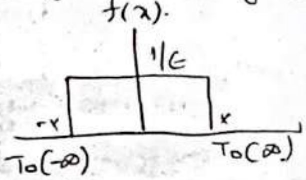
$$\lim_{\epsilon \to 0} g(\omega) = \lim_{\epsilon \to 0} \sqrt{\frac{2}{\pi}} \cdot \frac{\sin \omega\epsilon}{\omega\epsilon} \quad \left(\frac{0}{0} \text{ form}\right)$$

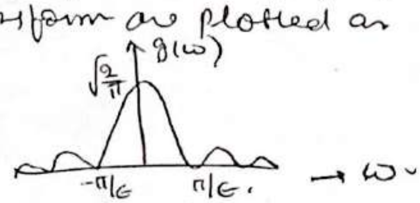
$$= \lim_{\epsilon \to 0} \sqrt{\frac{2}{\pi}} \, \frac{\frac{\partial}{\partial \epsilon}(\sin \omega\epsilon)}{\frac{\partial}{\partial \epsilon}(\omega\epsilon)} = \lim_{\epsilon \to 0} \sqrt{\frac{2}{\pi}} \cdot \frac{\omega \cos \omega\epsilon}{\omega}$$

$$= \sqrt{\frac{2}{\pi}}$$

Thus $g(\omega) \to \sqrt{\frac{2}{\pi}}$ as $\epsilon \to 0$, while $f(x) \to \infty$ as $x \to 0$

The fn and its fourier transform are plotted as





Example! - Find the Fourier Transform of the Gaussian distribution function

$$f(x) = N e^{-\alpha x^2}$$

where N and α are constants

Solution !- The Fourier Transform of the function $f(x)$ is given by

$$g(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x)\, e^{-i\omega x}\, dx.$$

$$\text{ie } g(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} N e^{-\alpha x^2} e^{-i\omega x}\, dx.$$

$$= \frac{N}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-(\alpha x^2 + i\omega x)}\, dx$$

$$= \frac{N}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\alpha\left(x^2 + \frac{i\omega}{\alpha} x\right)}\, dx.$$

$$= \frac{N}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\alpha\left[x^2 + \frac{i\omega}{\alpha} x + \left(\frac{i\omega}{2\alpha}\right)^2\right]}\, e^{\alpha\left(\frac{i\omega}{2\alpha}\right)^2}\, dx$$

$$= \frac{N}{\sqrt{2\pi}} e^{-\frac{\omega^2}{4\alpha}} \int_{-\infty}^{+\infty} e^{-\alpha\left(x + \frac{i\omega}{2\alpha}\right)^2}\, dx$$

$$= \frac{N}{\sqrt{2\pi}} e^{-\omega^2/4\alpha} \int_{-\infty}^{+\infty} e^{-\alpha y^2}\, dy \qquad \text{Subs } x + \frac{i\omega}{2\alpha} = y$$

$$= \frac{N}{\sqrt{2\pi}} e^{-\omega^2/4\alpha} \sqrt{\frac{\pi}{\alpha}} = \frac{N}{\sqrt{2\alpha}} e^{-\omega^2/4\alpha}$$

Example 3 Find the Fourier transform of $e^{-|t|}$

Solution $\quad F.T.\, e^{-|t|} = g(\omega)$ say

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-|t|} e^{-i\omega t}\, dt$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{\infty}^{0} e^{-|t|} e^{-i\omega t}\, dt + \int_{0}^{\infty} e^{-|t|} e^{-i\omega t}\, dt \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{0} e^{t} e^{-i\omega t}\, dt + \int_{0}^{\infty} e^{-t} e^{-i\omega t}\, dt \right]$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} e^{t(1-i\omega)} dt + \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-t(1+i\omega)} dt$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{e^{t(1-i\omega)}}{1-i\omega} \right]_{-\infty}^{0} + \frac{1}{\sqrt{2\pi}} \left[\frac{e^{-t(1+i\omega)}}{1+i\omega} \right]_{0}^{\infty}$$

$$= \frac{1}{\sqrt{2\pi}} \left(\frac{1}{1-i\omega} + \frac{1}{1+i\omega} \right) = \frac{1}{\sqrt{2\pi}} \cdot \frac{2}{1+\omega^2} = \sqrt{\frac{2}{\pi}} \left(\frac{1}{1+\omega^2} \right)$$

Example Find the sine transform of $\frac{e^{-ax}}{x}$.

Solution. The sine transform of function $f(x)$

$$g_s(\omega) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin \omega x \, dx$$

$$= \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{e^{-ax}}{x} \sin \omega x \, dx$$

$\frac{\sin \omega x}{x} = \frac{0}{0}$ type

Differentiating w.r.t ω, we get

$\therefore$ take diff.

$$\frac{d g_s(\omega)}{d\omega} = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{e^{-ax}}{x} x \cos \omega x \, dx$$

$$= \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-ax} \cos \omega x \, dx = \sqrt{\frac{2}{\pi}} \frac{a}{a^2+\omega^2}$$

Since $\int_0^\infty e^{-ax} \cos \omega x \, dx = \frac{a}{a^2+\omega^2}$.

Integrating we get

$$g_s(\omega) = \sqrt{\frac{2}{\pi}} \tan^{-1}\left(\frac{\omega}{a}\right) + A, \text{ A being constant of integration}$$

For $\omega = 0$, this gives $g_s(\omega) = g_s(0) = A$

But $g_s(\omega) = 0$ for $\omega = 0$, thereby giving $A = 0$.

Hence the required Fourier sine transform

$$= g_s(\omega) = \sqrt{\frac{2}{\pi}} \tan^{-1}\left(\frac{\omega}{a}\right)$$

Example:- Find the cosine transform of a function of x which is unity for $0 < x < a$ and zero for $x \geq a$. What is the function whose cosine transform is $\sqrt{\frac{2}{\pi}} \frac{\sin ap}{p}$.

Soln Given $f(x) = \begin{cases} 1 & \text{for } 0<x<a \\ 0 & x \geq a \end{cases}$

(i) The Fourier cosine transform of $f(x)$ is given by

$$g_c(\omega) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos\omega x \, dx$$

$$= \sqrt{\frac{2}{\pi}} \left[\int_0^a f(x)\cos\omega x \, dx + \int_a^\infty f(x)\cos\omega x \, dx \right]$$

$$= \sqrt{\frac{2}{\pi}} \left[\int_0^a 1\cdot\cos\omega x \, dx + \int_a^\infty 0\cdot\cos\omega x \, dx \right.$$

$$= \sqrt{\frac{2}{\pi}} \cdot \frac{\sin\omega a}{\omega}$$

(ii) Given $g(p) = \sqrt{\frac{2}{\pi}} \sin\frac{ap}{p}$ or $g(\omega) = \sqrt{\frac{2}{\pi}} \frac{\sin a\omega}{\omega}$

The Fourier inverse cosine transform is

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty g(\omega)\cos\omega x \, d\omega$$

$$= \sqrt{\frac{2}{\pi}} \int_0^\infty \sqrt{\frac{2}{\pi}} \frac{\sin\omega a}{\omega} \cos\omega x \, dx$$

$$= \frac{1}{\pi} \int_0^\infty \frac{2\sin\omega a \cos\omega x}{\omega} \, d\omega$$

$$= \frac{1}{\pi} \int_0^\infty \left[\frac{\sin(a+x)\omega + \sin(a-x)\omega}{\omega} \right] d\omega$$

$$= \frac{1}{\pi} \left[\int_0^\infty \left\{ \frac{\sin(a+x)\omega}{\omega} d\omega + \frac{\sin(a-x)}{\omega} \right\} d\omega \right.$$

$$= \begin{cases} \frac{1}{\pi} \left\{ \frac{\pi}{2} + \frac{\pi}{2} \right) & \text{if } x<a \\ \frac{1}{\pi} \left(\frac{\pi}{2} - \frac{\pi}{2} \right) & \text{if } x>a \end{cases} \qquad \left[\text{since } \int_0^\infty \frac{\sin\alpha x}{x} dx = \frac{\pi}{2} \text{ for } \alpha>0 \right]$$

$$\therefore f(x) = \begin{cases} 1 & \text{if } x<a \\ 0 & \text{if } x>a \end{cases}$$

(3) **Example:-** Find the Fourier transform of

$$F(x) = \begin{cases} 1, & |x|<a \\ 0, & |x|>a. \end{cases}$$

Hence evaluate $\int_0^\infty \frac{\sin^2 x}{x^2} dx$.

We have $g(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) e^{-i\omega x} dx = \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} e^{-i\omega x} dx$

$$= \frac{1}{\sqrt{2\pi}} \frac{e^{-i\omega x}}{-i\omega} \Big|_{-a}^{a} = \frac{1}{\sqrt{2\pi}} \left[\frac{e^{i\omega a} - e^{-i\omega a}}{i\omega} \right] = \sqrt{\frac{2}{\pi}} \frac{\sin \omega a}{\omega}$$

for $n \neq 0$.

For $n=0$, $g(\omega) = \sqrt{\frac{2}{\pi}} \lim_{n \to 0} \frac{\sin \omega a}{\omega} = \sqrt{\frac{2}{\pi}} \lim_{n \to 0} \frac{na - \frac{1}{\lfloor 3} n^3 a^3 + \cdots}{n}$

$$= \sqrt{\frac{2}{\pi}}\, a.$$

Now using Parseval's identity, we find

$$\int_{-a}^{a} 1^2 dx = \int_{-\infty}^{+\infty} \frac{2}{\pi} \frac{\sin^2 \omega a}{\omega^2} d\omega$$

$$\frac{2}{\pi} \int_{-\infty}^{+\infty} \frac{\sin^2 \omega a}{\omega^2} d\omega = 2a$$

or $$\int_{-\infty}^{+\infty} \frac{\sin^2 \omega a}{\omega^2} d\omega = \pi a,$$

or $$\int_{0}^{\infty} \frac{\sin^2 \omega a}{\omega^2} = \frac{\pi}{2} a.$$

Fourier transform of functions of two or three variables.

(i) Function of two variables :- Let $f(x,y)$ be a function of two variables x and y. Keeping y constant its Fourier transform w.r.t x is given by

$$g_1(u,y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x,y) e^{-iux} dx \quad \text{—(1)}$$

Then considering y variable, the Fourier Transform of $g_1(u,y)$ is

$$g(u,v) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} g_1(u,y) e^{-ivy} dy \quad \text{—(2)}$$

Subs. (1) in (2), we get the Fourier transform of $f(x,y)$ as

$$g(u,v) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x,y) e^{-i(ux+vy)} dx\, dy \quad \text{—(3)}$$

Also keeping v constant, the Fourier inverse transform of $g(x,y)$ is

$$f_1(x,v) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} g(u,v) e^{iux} du \quad \text{—(4)}$$

Then considering y variable, the Fourier inverse transform of $f_1(x,v)$ is

$$f(x,y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f_1(x,v) e^{ivy} dv \quad -(5)$$

Substituting (4) in (5), we get two dimensional Fourier inverse transform of $g(u,v)$ as

$$f(x,y) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} g(u,v) e^{i(ux+vy)} du\, dv \quad -(6)$$

(ii) Function of three variables :- Let $f(x,y,z)$ be the function of three variables. Proceeding exactly as above we get for Fourier transform of $f(x,y,z)$ as

$$g(u,v,\omega) = \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x,y,z) e^{-i(ux+vy+\omega z)} dx\,dy\,dz \quad -(7)$$

and the three dimensional Fourier inverse transform is

$$f(x,y,z) = \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} g(u,v,\omega) e^{i(ux+vy+\omega z)} du\,dv\,d\omega \quad -(8)$$

The following properties of two dimensional Fourier transform can be verified :- Let $g(u,v)$ be the Fourier transform of $f(x,y)$

1. $F.T.[f^*(x,y)] = g^*(-u,-v)$ — (9)
2. $F.T.\left[f(x-x_0, y-y_0)\right] = g(u,v)\, e^{-i(ux_0+vy_0)}$ — (10)
3. $F.T.\left[f(x,y)\, e^{i(u_0x+v_0y)}\right] = g(u-u_0, v-v_0)$ — (11)
4. $F.T.\left[f(ax,by)\right] = \frac{1}{|ab|} g\left(\frac{u}{a}, \frac{v}{b}\right)$ — (12)
5. $F.T.\left[\frac{\partial f}{\partial x}\right] = iug$, $F.T.\left[\frac{\partial f}{\partial y}\right] = ivg$ — (13)

   $F.T.\left[\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right] = -(u^2+v^2)\, g(u,v)$ — (14)

   $F.T.\left[x f(x,y)\right] = i\frac{\partial g}{\partial u}$; $F.T.\left[y f(x,y)\right] = i\frac{\partial g}{\partial v}$

   $F.T.\left[xy\, f(x,y)\right] = -\frac{\partial^2 g}{\partial u \partial v}$
6. $g(u,v) = g_1(u)\, g_2(v)$

   where $g_1(u) = F.T.[f_1(x)]$, $g_2(v) = F.T.[f_2(y)]$

   and $f(x,y) = f_1(x) f_2(y)$

Example:- Find the Fourier Transform of e^{-r^2/a^2} where a is a constant and $r=\sqrt{x^2+y^2+z^2}$

Solution:- Here fn f is the function of three variables (x,y,z)

i.e. $f(x,y,z) = e^{-r^2/a^2} = \frac{3}{(2\pi)^{3/2}} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-(x^2+y^2+z^2)} e^{-i(ux+vy+wz)}\,dx\,dy\,dz$

$$= \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{+\infty} e^{-x^2/a^2} e^{-iux}\,dx \int_{-\infty}^{+\infty} e^{-y^2/a^2} e^{-ivy}\,dy \int_{-\infty}^{+\infty} e^{-z^2/a^2} e^{-iwz}\,dz \quad \text{— (1)}$$

Now $\int_{-\infty}^{+\infty} e^{-x^2/a^2} e^{-iux}\,dx = \int_{-\infty}^{+\infty} e^{-(x^2+iua^2x)/a^2}\,dx$

$$= \int_{-\infty}^{+\infty} e^{-(1/a^2)\{x^2+iua^2x+(ia^2u/2)^2\}} \cdot e^{i^2a^2u^2/4}\,dx$$

$$= e^{-a^2u^2/4} \int_{-\infty}^{+\infty} e^{-\{(x+ia^2u/2)^2/a^2\}}\,dx$$

$$= e^{-a^2u^2/4} \int_{-\infty}^{+\infty} e^{-p^2/a^2}\,dp \quad \left(\text{subs. } x+\frac{ia^2u}{2}=p\right)$$

$$= e^{-a^2u^2/4} \sqrt{\frac{\pi}{1/a^2}} \quad \left[\text{since } \int_{-\infty}^{+\infty} e^{-\alpha p^2} = \sqrt{\frac{\pi}{\alpha}}\right]$$

$$= e^{-a^2u^2/4}\, a\sqrt{\pi}$$

Similarly, $\int_{-\infty}^{+\infty} e^{-y^2/a^2} e^{-ivy}\,dy = e^{-a^2v^2/4}\, a\sqrt{\pi}$

and $\int_{-\infty}^{+\infty} e^{-z^2/a^2} e^{-iwz}\,dz = e^{-a^2w^2/4}\, a\sqrt{\pi}$

Hence eqn (1) gives

$$g(u,v,w) = \frac{1}{(2\pi)^{3/2}} e^{-a^2u^2/4}\, a\sqrt{\pi}\; e^{-a^2v^2/4}\, a\sqrt{\pi}\; e^{-a^2w^2/4}\, a\sqrt{\pi}$$

$$= \frac{a^3}{(2)^{3/2}} e^{-(u^2+v^2+w^2)a^2/4}.$$

Finite Fourier Transforms

(i) Finite Fourier Sine Transform:- If $f(x)$ is an odd function of x in the interval $(-l,l)$, we have from Fourier series expansion

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} \quad \text{—(1)}$$

where $$b_n = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}\,dx \quad \text{—(2)}$$

The finite Fourier sine transform for function $f(x)$ is

Simple Applications of Fourier Transforms

(1) Evaluation of Integrals :- Using Fourier transforms certain integrals may be evaluated. For example to evaluate integrals

$$\int_0^\infty \frac{\cos nx}{a^2+n^2}\,dn \quad \text{and} \quad \int_0^\infty \frac{n \sin nx}{a^2+n^2}\,dn \text{ ; let us consider}$$

$$I_1 = \int_0^\infty e^{-ax} \cos nx \, dx \quad —(1)$$

$$\text{and} \quad I_2 = \int_0^\infty e^{-ax} \sin nx \, dx \quad —(2)$$

Integrating by parts, we get

$$I_1 = \left[-\frac{1}{a} e^{-ax} \cos nx\right]_0^\infty - \frac{1}{a}\int_0^\infty e^{-ax} \sin nx \, dx \quad —(3)$$

$$= \frac{1}{a} - \frac{n}{a} I_2$$

Similarly $I_2 = \left[-\frac{1}{a} e^{-ax} \sin nx\right]_0^\infty + \frac{n}{a}\int_0^\infty e^{-ax} \cos nx \, dx = \frac{n}{a} I_1$ —(4)

Solving (3) and (4), we get

$$I_1 = \frac{a}{a^2+n^2} \quad \text{and} \quad I_2 = \frac{n}{a^2+n^2} \quad —(5)$$

Now choosing $f(x) = e^{-ax}$, the cosine and sine transforms of $f(x)$ are

$$g_c(n) = \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-ax} \cos nx \, dx = \sqrt{\frac{2}{\pi}} \frac{a}{a^2+n^2} \quad —(6)$$

$$\text{and} \quad g_s(n) = \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-ax} \sin nx \, dx = \sqrt{\frac{2}{\pi}} \frac{n}{a^2+n^2} \quad —(7)$$

So that the Fourier inverse transformations yield

$$f(x) = e^{-ax} = \sqrt{\frac{2}{\pi}} \int_0^\infty g_c(n) \cos nx \, dx = \frac{2}{\pi}\int_0^\infty \frac{a}{a^2+n^2} \cos nx \, dn \quad —(8)$$

$$\text{and} \quad f(x) = e^{-ax} = \sqrt{\frac{2}{\pi}} \int_0^\infty g_s(n) \sin nx \, dn = \frac{2}{\pi}\int_0^\infty \frac{n}{a^2+n^2} \sin nx \, dn \quad —(9)$$

Equations (8) and (9) lead to the integrals

$$\int \frac{\cos nx}{a^2+n^2}\,dn = \frac{\pi}{2a} e^{-ax}$$

$$\int_0^\infty \frac{n \sin nx}{a^2+n^2}\,dn = \frac{\pi}{2} e^{-ax}$$

(2) Solution of Boundary Value Problems :- The Fourier transforms may be applied to solve certain boundary