Frame of Reference:— A system of co-ordinate axes which (1)
defines the position of a particle or an event in two or three
~~dimension~~ dimensional space is called a frame of reference.
The essential requirement of a frame of reference is that it
should be rigid. The frame of reference must be at rest
w.r. to him (observer).

for complete identification of an event in a reference
frame i.e. for determination of exact location as well as
the exact time of its occurrence we must in addition to x, y, z
have another coordinate i.e. 't'.

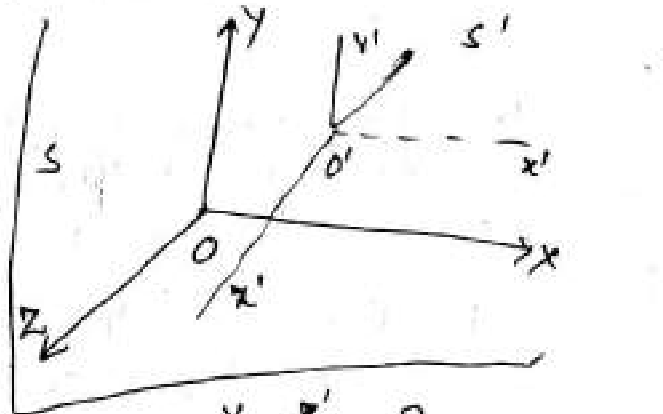
A reference frame with coordinate x, y, z, t is called
Space-time frame, these four coordinate are called space-time.

Inertial frames of reference:— (Newtonian or

The first two law of Newton do not always hold good in each &
every frame of references. For a body at rest in one refrence frame
may appear to be moving in a circle in an another frame of references.
Hence frame of references in which two law of motion hold good
is called Newtonian, inertial or Galilean frame of reference.

In inertial frame of reference

acceleration $a = \frac{d^2r}{dt^2} = 0 \Rightarrow \frac{dr}{dt} =$



$S \rightarrow$ an inertial frame at rest

S' — another moving with velocity v

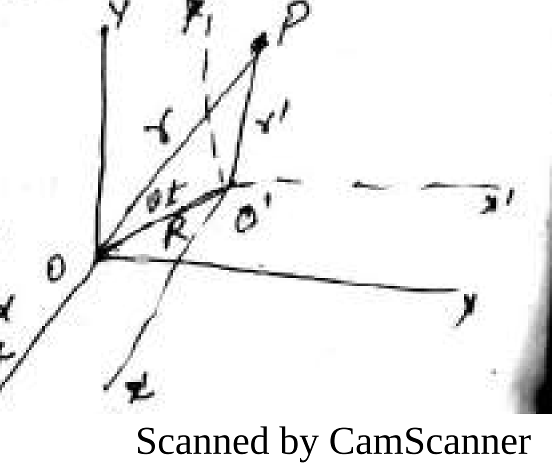
r & r' — position vector of particle P.

$r = R + r'$

$r' = r - R = r - vt$

$\frac{d^2r'}{dt^2} = \frac{d^2r}{dt^2}$, $v =$ const^n

i.e. acceleration of particle is ~~constant~~ same
in two frame of reference $\Rightarrow$ If the particle
be at rest in the inertial frame S, it would
also appear to be at rest in frame S'.



... collision in C-system ...

Conclusion :— all frames of reference moving with a constant velocity an inertial frame, are also inertial frame of reference.

Ex :— earth, sun, star,

Absolute frame of references :—

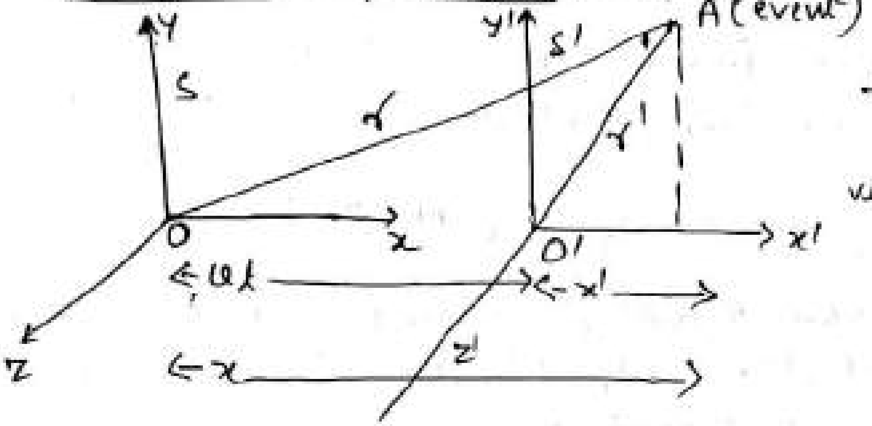
There must be exist a fundamental or absolute frame of reference w.r.t. to which all motions must be measured.

ex :— Star.

Galilean Transformation :—

Eqn relating the two set of co-ordinates of the event is two system (inertial) are called transformation.

(1) Transformation of Position :—



S & S' – two inertial frame of references.
S' is moving with a relative velocity v at +ve x dirn.

Let some event occur at A. According to observer O in S-frame determines the position of event by co-ordinates x, y, z and observer O' is by x', y', z'.

$$x' = x - vt, \quad y' = y, \quad z' = z$$

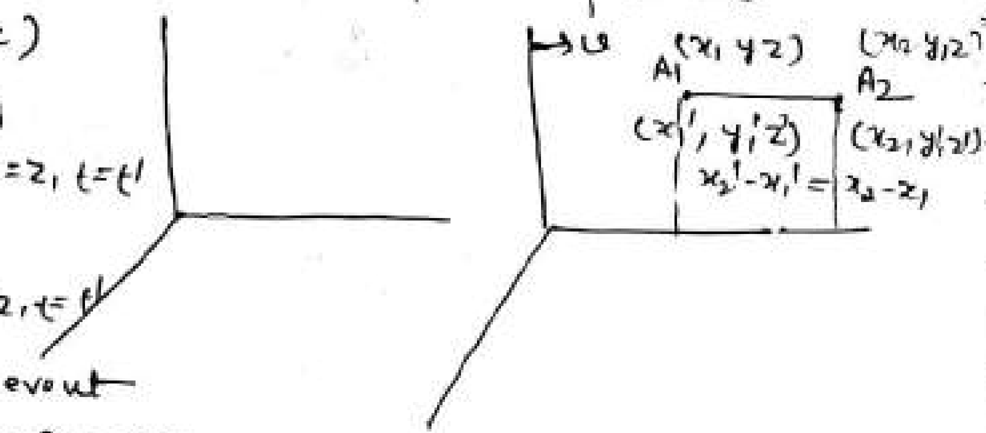
we assume that time is independent any frame of reference.

$$t = t'$$

(2) Transformation of length :—

Let there are two events occuring simultaneously at A_1 & A_2 some distance apart, along x-axis.

(x_1, y, z), (x_2, y, z)



Transformation eqn for A_1

$x_1' = x_1 - vt, \ y' = y, \ z' = z, \ t = t'$

for A_2

$x_2' = x_2 - vt, \ y' = y, \ z' = z, \ t = t'$

hence distance betn two event

$$x_2' - x_1' = \Delta x' = x_2 - x_1 = \Delta x$$

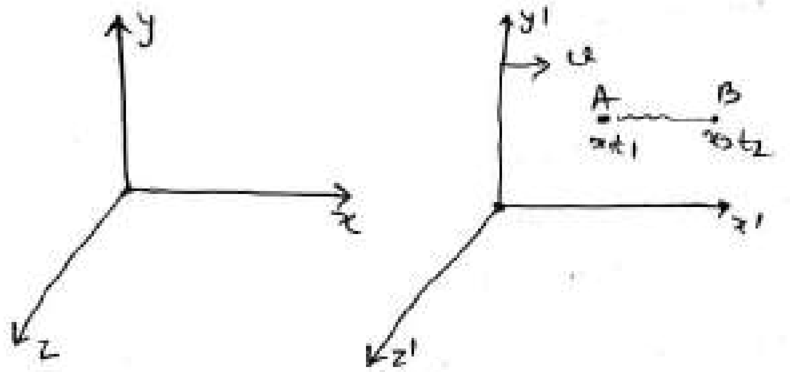
the distance betn two event, measured by the observer in either frame of references is same, irrespective of the value of v & t.

$\Delta x = L$ length of rod means

thus, length of rod is invariant in Galilean Transformation.

Transformation of velocity :—

Let u & u' be the velocity of an event made by the two observer in frame S & S'. We consider a displacement in +ve x-dirn & the time interval for it. If (x_1, t_1) and (x_2, t_2) be the coordinate of initial and final event



$$u = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$= \frac{(x_2' + v t_2) - (x_1' + v t_1)}{t_2' - t_1'}$$

$$= \frac{x_2' - x_1'}{t_2' - t_1'} + v \frac{t_2' - t_1'}{t_2' - t_1'}$$

$$= \frac{\Delta x'}{\Delta t'} + v$$

$$= u' + v$$

$$\boxed{u' = u - v}$$

Again we show that

$$x' = x - vt$$

$$\frac{dx'}{dt} = \frac{dx}{dt} - v = \boxed{u' = u - v}$$

velocity measured by the observer in the two frame of reference are not same, velocity is not invariant to Galilean Transformation.

(4) Transformation of acceleration :—

Consider a & a' acceleration measured by observer in their respective frame of reference.

$$a = \frac{du}{dt}, \quad \& \quad a' = \frac{du'}{dt}$$

Since $u' = u - v$

$$\frac{du'}{dt} = \frac{du}{dt} - 0 \Rightarrow \boxed{a' = a}$$

i.e. accn is invariant to Galilean transformation.

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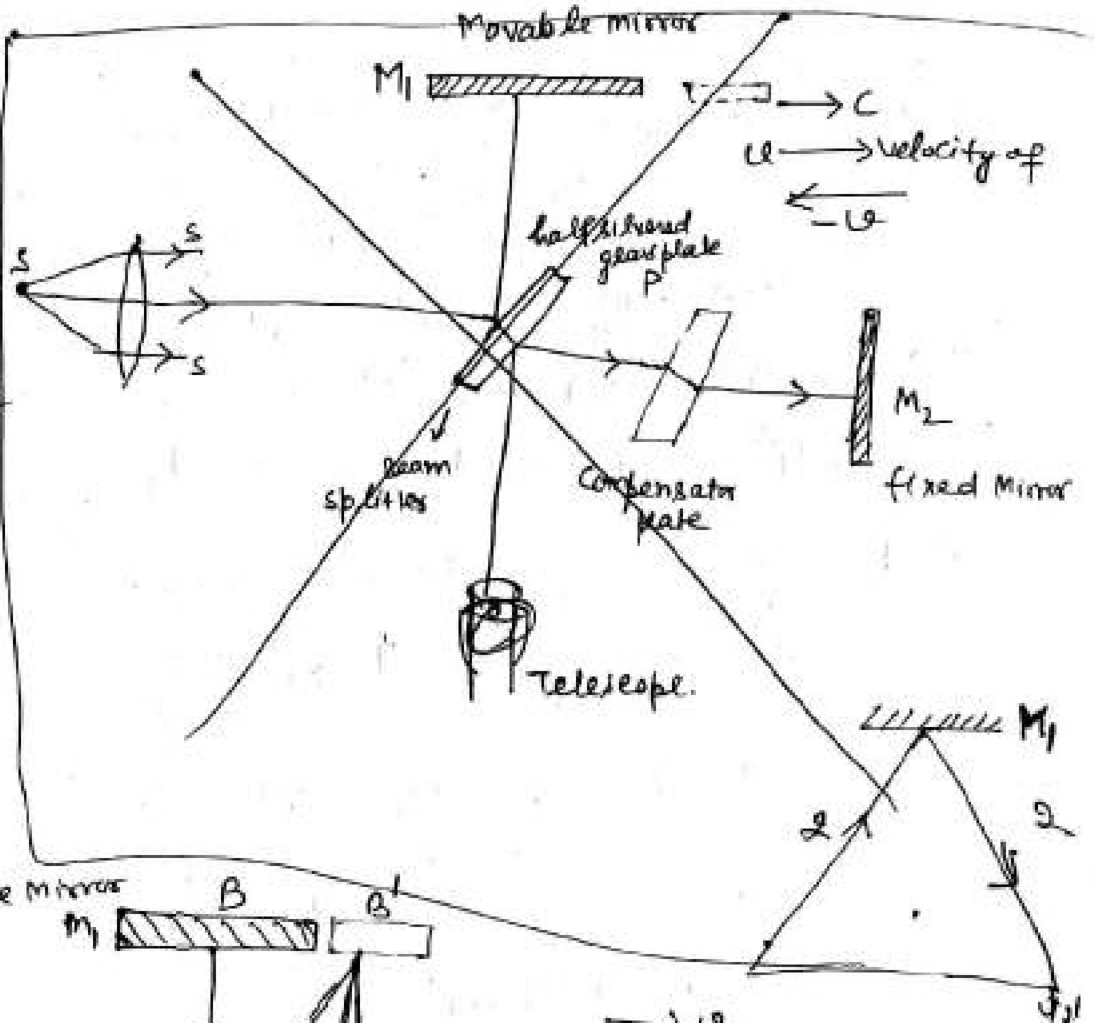
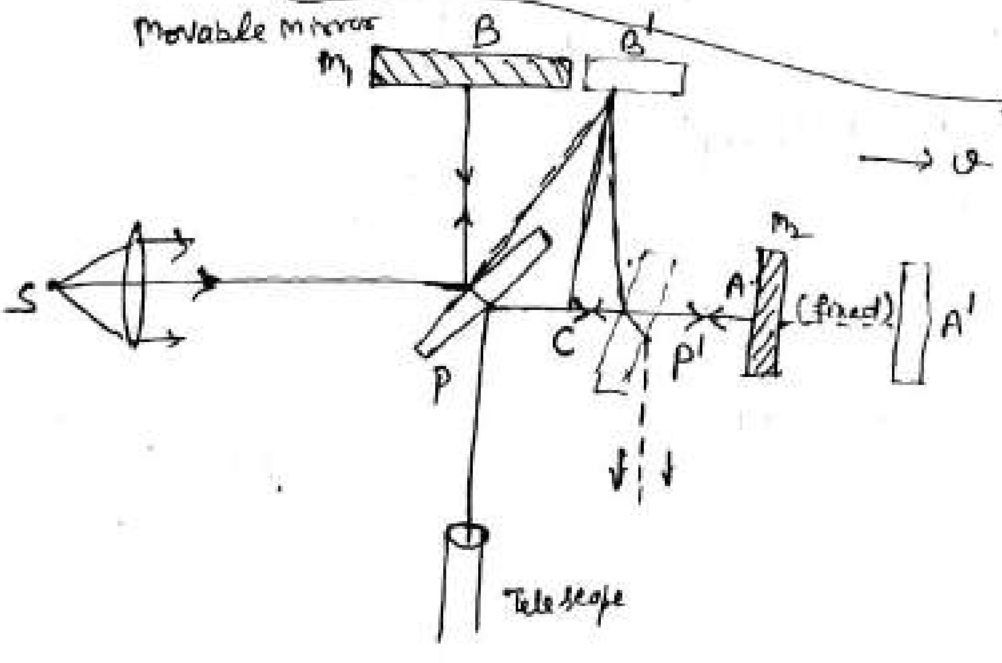
Michelson - Morley experiment :- (search of fundamental frame of Ref.)

In 1887 Michelson and Morley set out to measure the relative velocity of earth with respect to the ether. The principle of the experiment lies in noting the shift in fringes in the Michelson interferometer due to the difference in time taken by light to travel along and oppositive dir^n of motion of earth. The time taken by a beam of light to travel along the dir^n of motion of earth is greater than that of travel distance oppositive to the dir^n of motion ~~of~~ the earth.

The purpose of this experiment ~~to~~ was to measure the time diff. from which velocity of ~~earth~~ earth relative to the ether.





One arm (PA) was pointed in the dirn of earth's motion the sun and the other (PB) was pointed ⊥ to this motion. assume that the velocity of apparatus (or earth) relative to fixed ether is v in dirn PA. The relative velocity of light ray along PA is $(c-v)$ and for returning ray $(c+v)$.

Let $PA = PB = d$

Time taken by light to travel from P to A $= \frac{d}{(c-v)}$

from A to P $= \frac{d}{c-v}$

total time taken by light to travel P to A and back

$$t = \frac{d}{c-v} + \frac{d}{c+v} = \frac{2cd}{c^2-v^2} = \frac{2d}{c}\left(1+\frac{v^2}{c^2}\right) \quad \text{——— (1)}$$

Now consider the ray moving upward from P to B. It will strike the mirror M_1 not at B but B' due to motion of earth. If t_1 time taken by ray from P to M_1 then

$PB' = ct_1$, & $BB' = vt_1$

Now $PB'P' = PB' + B'P' = 2PB'$ since $PB' = B'P'$

Now $(PB')^2 = (PC)^2 + (CB')^2$

$(ct_1)^2 = (vt_1)^2 + d^2$ $\quad \therefore t_1 = \frac{d}{\sqrt{c^2-v^2}}$

total time taken by ray to travel the whole path PB'P'

$$t' = 2t_1 = \frac{2d}{\sqrt{c^2-v^2}} = \frac{2d}{c}\left(1+\frac{v^2}{2c^2}\right) \quad \text{——— (2)}$$

Clearly that $t > t'$, then time diff. $= t - t' = \frac{dv^2}{c^3}$

then path difference $= c \times \Delta t = \frac{dv^2}{c^2}$

If operatus is turned through 90°.

Mirror M_1 & M_2 exchange their roles. the total shift will be twice of above result $= 2\frac{dv^2}{c^2} = n\lambda$

$n = 0.4$

$l = 1100$ cm
$\lambda = 5500$ A°
$v = 3\times10^6$ cm/sec
$c = 3\times10^{10}$ cm/sec

But in experiment no displacement of the fringe was found. They repeated the experiment at different points of earth & measuring the shift in fringe, but no result found.

negative result suggest that it is impossible to measure (2)
speed of earth relative to ~~ea~~ ether.

Explanation of negative result:—

(1) No relative motion betn earth and ether. $\Rightarrow v=0 \Rightarrow$ then $\boxed{t=t'}$

(2) Lorentz and ~~Fitzg~~ Fitzgerald suggest that there was contraction of bodies along the dirn of their motion, through the ether.
the contracted length is $l_0\sqrt{1-v^2/c^2}$.
we replaced d by $d\sqrt{1-v^2/c^2}$ in (1) then t and t' are same.

(3) Einstein. He concluded that the velocity of light in a space is universal constant.

Ether:—

* According to Michelson-Morley eqn (8) gives the difference of distance travelled by light betn ~~para~~ parallel and transverse dirn when operatus rotate through 90°

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~~T~~heory of Relativity: — It refers ~~only~~ two theories:

1- Special relativity 2. General Relativity.

Special Relativity: — It is proposed by Einstein in 1905:

1. The law of Physics is same in all inertial frame of reference.
2. The speed of light in a vaccum is a universal constant, which is indepent of the motion of light source.

Intestial frame of Refrences: — In which newton first and second law of motion are valid.

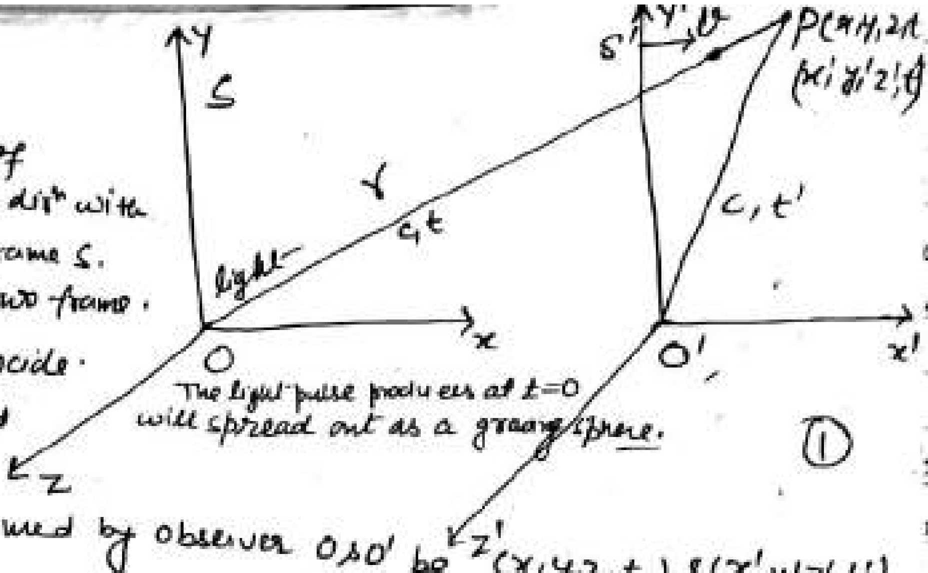
Special: —

(I) Time dilation — Moving clocks tick slower than an observer's 'stationary' clock.

(II) Length contraction: — object are observed to be small in the direction that they are moving with respect to observer.

(III) Relativity of simultaneity: — Two events that appear simultaneous to an observer A will not be simultaneous to an observer B if B is moving with respect to A.

...tz/ Transformation



...t S and S' be the two frame of ...eference. S' is moving along +ve x dir^n with constant velocity υ relative to frame S.
Let t & t' time recorded ~~by~~ in two frame.
at $t = t' = 0 \Rightarrow O$ & O' be concide.

Let a light placed at O is emitted at $t=0$, when light reaches at P, the position & time measured by observer O & O' be (x, y, z, t) & (x', y', z', t')
If c is the velocity of light.

Let the time taken by light signal to travelling the distance OP in frame S is

$$t = \frac{OP}{c} = \frac{(x^2+y^2+z^2)^{1/2}}{c} \Rightarrow \boxed{x^2+y^2+z^2 = c^2t^2} \quad \text{——} (1)$$

according to Einstein theory the velocity of light is also c in frame C.
then time required by light to travell a distance O'P in S' is

$$t' = \frac{O'P}{c} = \frac{(x'^2+y'^2+z'^2)^{1/2}}{c} \Rightarrow \boxed{x'^2+y'^2+z'^2 = c^2t'^2} \cdots (2)$$

Now Galilean transformation gives

$$x' = x - \upsilon t, \quad y' = y, \quad z' = z, \quad t' = t$$

then from eq^n (2)

$$(x-\upsilon t)^2 + y^2 + z^2 = c^2t^2 \quad \text{or} \quad x^2 - \underline{2x\upsilon t + \upsilon^2t^2} + y^2 + z^2 = c^2t^2 \quad \text{——} (3)$$

eq^n (3) is not in agreement with eq^n (1) due to extra term $(-2x\upsilon t + \upsilon^2t^2)$
$\Rightarrow$ Thus the Galilean transformation fails if ~~the~~ we assume velocity of light is constant.

In eq^n (3) & (1) the terms of y & z are in agreement i.e. $y' = y$, $z = z'$
Hence the extra term $(-2x\upsilon t + \upsilon^2t^2)$ indicates that the transformation in x & t should be modified so that extra term cancelled.
Let $OO' = \upsilon t$

$$x' = \alpha(x - \upsilon t)$$

and t' is different from t and may be depending on x, so that we also assume that

$\boxed{t' = \alpha'(t + fx)}$, Here α, α' & f are constant to be determined.

for Galilean transformation $(\alpha = \alpha' = 1, f = 0)$
substituting for x', y', z', & t' in eq^n (2)

$$\alpha^2(x-\upsilon t)^2 + y^2 + z^2 = c^2\alpha'^2(t + fx)^2$$

$$x^2(\alpha^2 - c^2\alpha'^2f^2) - 2xt(\upsilon\alpha^2 + c^2\alpha'^2 f) + y^2 + z^2 = c^2t^2\left(\alpha'^2 - \frac{\alpha^2\upsilon^2}{c^2}\right) \quad \text{—} (IV)$$

eq^n (IV) reduces to (1) if

$$\alpha^2 - c^2\alpha'^2f^2 = 1, \quad \alpha^2\upsilon + c^2\alpha'^2 f = 0, \quad \alpha'^2 - \frac{\alpha^2\upsilon^2}{c^2} = 1$$

$$\boxed{\alpha = \alpha' = \frac{1}{\sqrt{1-v^2/c^2}}} \;\&\; \boxed{f = -\frac{v}{c^2}}$$

$$\therefore \boxed{x' = \frac{x - vt}{\sqrt{1-v^2/c^2}},\ y' = y,\ z' = z,\ t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-v^2/c^2}}}$$

If $v << c \Rightarrow \frac{v}{c} \to 0$ $x' = x - vt,\ y' = y,\ z' = z,\ t' = t \Rightarrow$ Galilean Trans

The Inverse transformation eqn:— S is moving with $-v$ velocity relative to S' along $-ve$ x dirn

$$\boxed{x = \frac{x' + vt}{\sqrt{1-v^2/c^2}},\ y = y',\ z = z',\ t = \frac{t' + \frac{vx}{c^2}}{\sqrt{1-v^2/c^2}}}$$

Ex:- Qn on page 20:—

Show that square of square of space-time interval betn two event is invariant under Lorentz transformation.

In frame S $\Rightarrow S_{12}^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 - c^2(t_2 - t_1)^2$

When $\boxed{S_{12}^2 = S_{12}'^2}$

start from $S_{12}'^2 = (x_1' - x_2'^2) + (y_1' - y_2'^2) + (z_1' - z_2')^2 - c^2(t_1' - t_1'^2)$

Consequence of ~~Lr~~ Lorentz Transformation:—

(1) Length Contraction:— the length of rod measured in frame of ref. at rest w.r. to observer is called proper length.

and coordinate of o rod in frame S is measured by x_1 & x_2 at same time t

$l = x_2 - x_1$

according to Lorentz transform

$$x_1' = \frac{x_1 - vt}{\sqrt{1-v^2/c^2}},\quad x_2 = \frac{x_2 - vt}{\sqrt{1-v^2/c^2}} \Rightarrow l_0 = x_2' - x_1' = \gamma(x_2 - x_1)$$

$$\Rightarrow \boxed{l = l_0\sqrt{1-v^2/c^2}} \qquad \Rightarrow l_0 > l$$

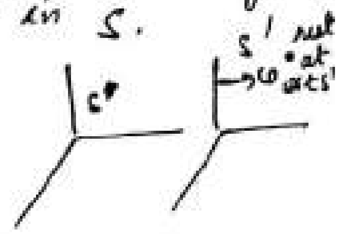
hence length of rod measured by observer O is smaller than its proper length.

(II) ex:— L_0^3 is rest volume of cube then what is observed volume ... If moving with velocity v.

The length measured by an observer in a frame, at rest relative to is called its proper length

(Ti)me Dilation:—

If two events occur at a given point x' in frame S' at time t_1' & t_2' as noted by clock carried by it S', and time t_1 & t_2 be noted in S.



Let time interval in $S' \Rightarrow \Delta t' = t_2' - t_1'$

& w.r.to. $S \Rightarrow \Delta t = t_2 - t_1$

according to Lorentz transformation

$$t_1 = \gamma\left(t_1' + \frac{v x'}{c^2}\right), \quad t_2 = \gamma\left(t_2' + \frac{v x'}{c^2}\right)$$

$$\Delta t = t_2 - t_1 \Rightarrow \boxed{\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}}} \qquad \boxed{\Delta t > \Delta t'}$$

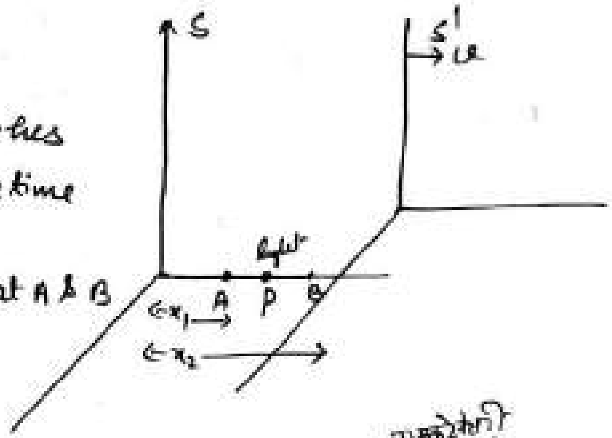
Thus, the time interval, measured in the frame S is ~~longer~~ larger than the time interval in the frame S', in which the event are occuring at a certain point x'. time interval $\Delta t'$ is proper time, because two event occuring at same point in S'.

$\Rightarrow$ Time dilation is independent of of dir^n of velocity depends only on magnitude.

Twin: A (on earth), B (on rocket) } $\Rightarrow$ after some time when B return back to earth, B appears younger than A.

③ Simultaneity:—

If light is emitted from a Point P & reaches at A & B simultaneously i.e. at the same time t in frame S (at rest)



The time interval bet^n event occur at A & B is $\Delta t = t - t = 0$

Now for observer t_1' & t_2' respectively.

$$\Delta t' = (t_2' - t_1') = \gamma \frac{v}{c}(x_2 - x_1) \neq 0$$

④ Transformation of velocity:— (velocity addition)

Let the co-ordinate of a particle in frame S (at rest) are (x, y, z, t) and in frame S' (moving with velocity v along +ve x dir^n), (x', y', z', t'), then component of velocity (u in S, u' in S') in two frame

$$u_x = \frac{dx}{dt}, \quad u_y = \frac{dy}{dt}, \quad u_z = \frac{dz}{dt} \quad \text{in } S$$

$$u_x' = \frac{dx'}{dt'}, \quad u_y' = \frac{dy'}{dt'}, \quad u_z' = \frac{dz}{dt'} \quad \text{in } S'$$

from Lorentz transformation

$x = \gamma(x' + vt')$, $y = y'$, $z = z'$, $t = \gamma\left(t' + \frac{vx'}{c^2}\right)$

$dx = \gamma(dx' + v\,dt')$, $dy = dy'$, $dz = dz'$, $dt = \gamma\left(dt' + \frac{v\,dx'}{c^2}\right)$

$$u_x = \frac{dx}{dt} = \boxed{\frac{u_x' + v}{1 + \frac{v u_x'}{c^2}} = u_x}, \quad u_y = \frac{dy}{dt} = \frac{u_y'}{\gamma\left(1 + \frac{v}{c^2}u_x'\right)}$$

$$u_z = \frac{u_z'}{\gamma\left(1 + \frac{v}{c^2}u_x'\right)}$$

The inverse transformation by putting ~~v instead of v~~ $-v$ ins. of v

$$u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}, \quad u_y' = \frac{u_y}{\gamma\left(1 - \frac{v u_x}{c^2}\right)}, \quad u_z' = \frac{u_z}{\gamma\left(1 - \frac{v u_x}{c^2}\right)}$$

This is relativistic law of addition of velocity.

If there is only one component of velocity along x-dirn then

$$\boxed{u = \frac{u' + v}{1 + \frac{vu}{c^2}}} \quad \& \quad \boxed{u' = \frac{u - v}{1 - \frac{uv}{c^2}}}$$

Conclusion: (i) If $u_x' << c$ $\Rightarrow u_x = u_x' + v$ Galilean Transformation.

(ii) $u_x' = c$ $\Rightarrow u_x = \frac{c + v}{1 + \frac{v.c}{c^2}} = c \Rightarrow$ velocity of light

particle has same velocity in frame S. $\Rightarrow$ velocity of light is same in all inertial frame of reference

* Two velocity 7·8C are inclined to each other at an angle of 30°. Find their resultant value.

Relativistic energy: Mass - energy Relation ($E = mc^2$)

Suppose a force $F = \frac{d}{dt}(mu)$ be acting on a particle of mass m, so that its kinetic energy increases. The gain in kinetic energy will be equal to work done on the particle. If force displace the particle a distance dr along its line of motion, the small gain in kinetic energy

$$dE_K = F\,dr = \frac{d}{dt}(mu)\,dr = u\,d(mu)$$

$$E_K = \int dE_K = \int_0^u u\,d(mu)$$

$$= u \cdot mu \Big|_0^v - \int_0^v mu\,du = mu^2 - \int_0^u \frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}\,u\,du$$

$$= \frac{m_0 u^2}{\sqrt{1-\frac{u^2}{c^2}}} + m_0 c^2\sqrt{1-\frac{u^2}{c^2}} - m_0 c^2$$

$$= \frac{m_0 c^2}{\sqrt{1-\frac{u^2}{c^2}}} - m_0 c^2 = mc^2 - m_0 c^2$$

$E_K = (m - m_0)c^2 = \Delta m c^2$ ⇒ where Δm is the increase in mass with increase in velocity.

thus gain in kinetic energy $\boxed{\Delta E_K = \Delta m c^2}$

the quantity $E_0 = m_0 c^2$ is called rest mass energy.

Total energy

E = Kinetic energy + Rest energy (E_0)

$= (m - m_0)c^2 + m_0 c^2$

$$\boxed{E = mc^2}$$

Let now

$$E = \frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}c^2 \quad \text{for } u \ll c \text{ then}$$

$$E_K = m_0 c^2\left(1 - \frac{u^2}{c^2}\right)^{-1/2} - m_0 c^2$$

$$= m_0 c^2\left[1 + \frac{1}{2}\frac{u^2}{c^2} + \frac{3}{8}\frac{u^4}{c^4} \cdots\right] - m_0 c^2$$

$= \frac{1}{2} m_0 u^2$ ⇒ classical result.

...s Energy Relation:—

(2)

Let $F = \frac{d}{dt}(mv)$ ... (1) but both mass & velocity are variable, then

$$F = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt} \quad \ldots (2)$$

If force displace the body through a distance dx then gain in kinetic energy

$$dE_K = F.dx = m\frac{dv}{dt}.dx + v\frac{dm}{dt}.dx$$

$$dE_K = mv\,dv + v^2 dm \quad \ldots (3)$$

According to law of variation of mass with velocity

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad \ldots (4)$$

$$m^2c^2 = m_0^2c^2 + m^2v^2$$

Differentiating $c^2\, 2m\,dm = m^2\, 2v\,dv + v^2\, 2m\,dm$

$$c^2 dm = mv\,dv + v^2 dm \quad \ldots (5)$$

from (3) & (5)

$$dE_K = c^2 dm$$

$$E_K = \int_0^{E_K} dE_K = c^2\int_{m_0}^{m} dm = c^2(m - m_0)$$

$$\boxed{E_K = mc^2 - m_0c^2}$$ This is relativistic formulla.

Total energy $E = E_K + m_0c^2$

$$\boxed{E = mc^2}$$

This is Einstein energy mass relation.

Let $E_K = mc^2 - m_0c^2$

for $v << c$

$$E_K = m_0c^2\left[\left(1-\frac{v^2}{c^2}\right)^{-1/2} - 1\right]$$

$v << c$

$$\boxed{E_K = \frac{1}{2}m_0c^2 \times \frac{v^2}{c^2} = \frac{1}{2}m_0v^2}$$

Classical result.

Relation betⁿ Total energy, rest energy and momentum

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1-v^2/c^2}} \quad \text{—} (1)$$

& $p = mv$

$$\Rightarrow v = \frac{p}{m} \quad \text{—} (2)$$

from (1) & (2)

$$E = \frac{m_0 c^2}{\sqrt{1-\left(\frac{p^2}{m^2c^2}\right)}} = \frac{m_0 c^2}{\sqrt{1-\frac{p^2c^2}{m^2c^4}}}$$

$$= \cancel{} = m_0 \frac{c^2}{\sqrt{\left[1-\frac{p^2c^2}{E^2}\right.}}$$

$$\boxed{E^2 = m_0^2 c^4 + p^2 c^2}$$

Ex: two velocity of .8c are inclined to each other at an angle 30°. Find their resultant value.

Ans:— velocity of $s' = .8c$

u_x': $u'\cos 30 = .8c \times \frac{\sqrt{3}}{2} = .4\sqrt{3}\,c$

$u_y' = u'\sin 30 = .4c$

$$u_x = \frac{u_x' + v}{1+\frac{u_x' v}{c^2}} = .96c$$

$$u_y = \frac{u_y'\sqrt{1-\frac{v^2}{c^2}}}{1+\frac{u_x' v}{c^2}} = .15c$$

$$u = \sqrt{u_x^2 + u_y^2} = 0.97c$$

$$\phi = \tan^{-1}\left(\frac{u_y}{u_x}\right)$$

Pair production:— when a $\gamma > 1.02$ MeV strike with nucleus

$\gamma \rightarrow e^- + e^+$

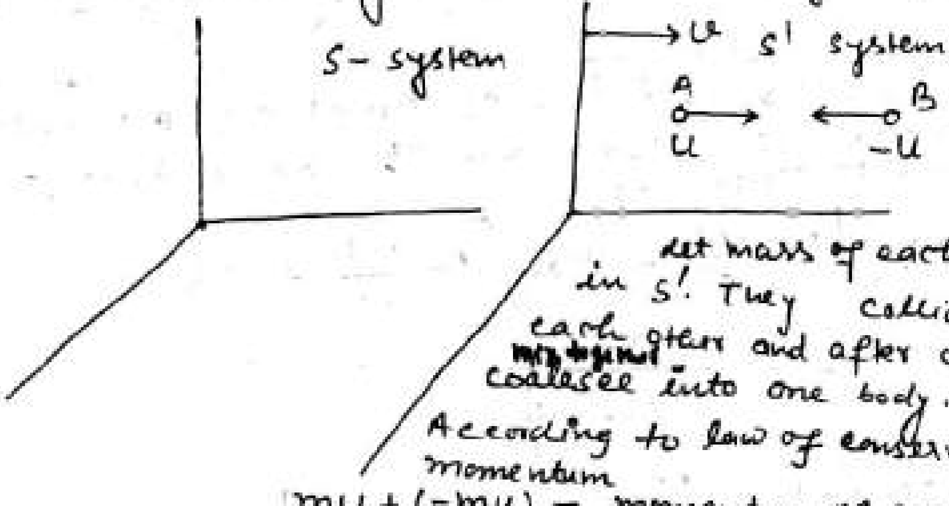
Pair Annihilation

$e^- + e^+ \rightarrow \gamma$

$$p_x' = \frac{p_x - \frac{vE}{c^2}}{\sqrt{1-v^2/c^2}}$$

$$E' = \frac{E - vp_x}{\sqrt{1-v^2/c^2}}$$

Relation of mass with velocity :- Consider two system S & S'



Let mass of each ball be m in S'. They collide with each other and after collision coalesce into one body.

According to law of conservation of momentum

$$mu + (-mu) = \text{momentum of coalesced mass} = 0$$

Let us now consider with respect to S. Let u_1 & u_2 be the velocities of ball relative to S, then

$$u_1 = \frac{u+v}{1+\frac{uv}{c^2}} \quad \text{--- (1)} \qquad u_2 = \frac{-u+v}{1-\frac{uv}{c^2}} \quad \text{--- (2)}$$

after collision the velocity of coalesced mass is v relative to S. Let mass of body be m_1 & m_2 w.r.to S

$$m_1u_1 + m_2u_2 = (m_1+m_2)v \quad \text{---- (3)}$$

$$m_1\left[\frac{u+v}{1+\frac{uv}{c^2}}\right] + m_2\left[\frac{-u+v}{1-\frac{uv}{c^2}}\right] = (m_1+m_2)v$$

after solving

$$\frac{m_1}{m_2} = \frac{1+uv/c^2}{1-\frac{uv}{c^2}} \quad \text{------ (4)}$$

also,

$$1-\frac{u_1^2}{c^2} = 1 - \frac{\left\{\frac{u+v}{c}\right\}^2}{\left\{1+\frac{uv}{c^2}\right\}^2}$$

$$1-\frac{u_1^2}{c^2} = \frac{\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1+\frac{uv}{c^2}\right)^2} \quad \text{----- (5)}$$

Similarly

$$1-\frac{u_2^2}{c^2} = \frac{\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{uv}{c^2}\right)^2} \quad \text{---- (6)}$$

(6) ÷ (5)

$$\frac{1+\frac{uv}{c^2}}{1-\frac{uv}{c^2}} = \frac{\sqrt{1-\frac{u_2^2}{c^2}}}{\sqrt{1-\frac{u_1^2}{c^2}}} \quad \text{--- (7)}$$

from (7) & (4)

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$$\therefore \frac{m_1}{m_2} = \frac{\sqrt{1-\frac{u_2^2}{c^2}}}{\sqrt{1-\frac{u_1^2}{c^2}}} \Rightarrow m_1\sqrt{1-\frac{u_1^2}{c^2}} = m_2\sqrt{1-\frac{u_2^2}{c^2}} \quad (8)$$

Since the L.H.S & R.H.S. of equation (8) are independent of one another, the above result can be true only if each is constant. Therefore

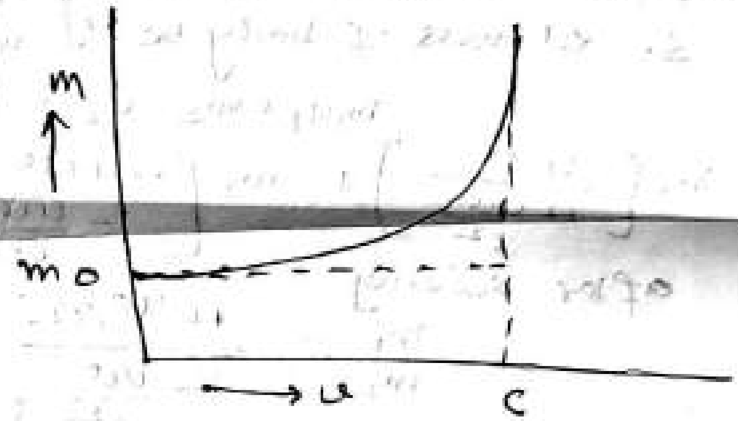
$$m_1\sqrt{1-\frac{u_1^2}{c^2}} = m_2\sqrt{1-\frac{u_2^2}{c^2}} = m_0$$

$$m_1 = \frac{m_0}{\sqrt{1-\frac{u_1^2}{c^2}}}$$

In general $\boxed{m = \frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}}$

If $u \to c \Rightarrow m \to \infty$

means an object travelling with c would have infinite mass.



when $u \ll c$

$$m = m_0\left(1-\frac{u^2}{c^2}\right)^{-1/2}$$

$$= m_0\left[1+\frac{1}{2}\left(\frac{u^2}{c^2}\right)+\frac{3}{8}\left(\frac{u^2}{c^2}\right)^2+\dots\right]$$

$\boxed{m = m_0}$ mass of particle = rest mass of particle.