Interference

- The general problem is to calculate the difference in optical path length, d, for two rays of light
  - Constructive interference: $kd = 0, \pm 2\pi, \pm 4\pi, \ldots$
  - Destructive interference: $kd = \pm\pi, \pm 3\pi, \ldots$
- Optical path length depends on geometry but also the index of refraction:

$$\ell = n_1\ell_1 + n_2\ell_2 + \cdots$$

- Resulting phase advance:

$$\delta = k\ell$$

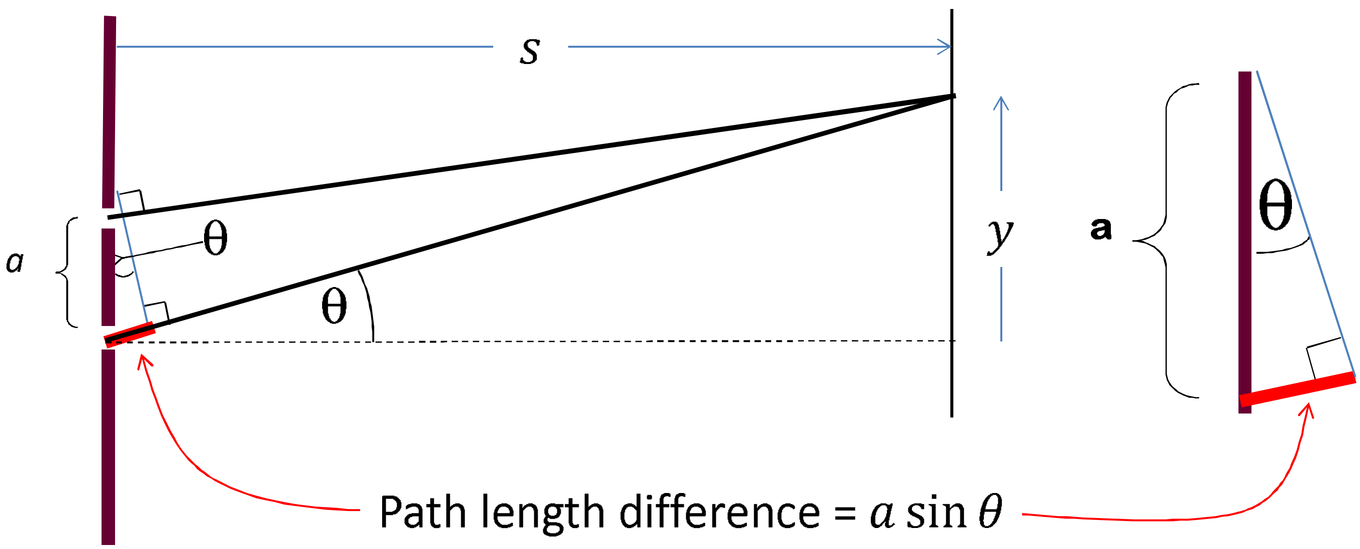
(where k is the wavenumber in vacuum)

- A reflection from the surface of a material with a larger index of refraction gives a phase shift of π.

Interference

- General expression for the intensity of two rays, each with equal intensity I_0:
$$I = 2I_0(1 + \cos\delta) = 4I_0 \cos^2\frac{\delta}{2}$$
- Phase difference: $\delta = \vec{k}_1 \cdot \vec{x} - \vec{k}_2 \cdot \vec{x} + \xi_1 - \xi_2$
- Usually we can choose $\xi_1 = 0$ for convenience
- If the sources are in phase, then we also have $\xi_2 = 0$.
- Maximum intensity occurs when $\delta = 0, \pm 2\pi, \ldots$
- This happens when the optical path lengths differ by $m\lambda$ where $m = 0,1,2, \ldots$

Young's Double-Slit Experiment



Constructive interference: $a\sin\theta = m\lambda$

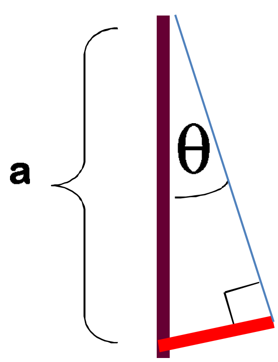
Destructive interference: $a\sin\theta = \left(m + \dfrac{1}{2}\right)\lambda$

where m = 0, ± 1, ± 2, ...

$\sin\boldsymbol{\theta} = \boldsymbol{y}/\boldsymbol{s}$ so there are maxima when $\boldsymbol{y} = \dfrac{\boldsymbol{m\lambda s}}{\boldsymbol{a}}$

Example Question

- If Young's double-slit experiment were performed under water ($n = 1.3$), at what values of y would the bright fringes appear?
  - The path lengths are the same, but the optical path lengths are different.



Constructive interference when

$$na \sin\theta = m\lambda$$

$$\sin\theta = \frac{y}{s}$$

$$y = \frac{m\lambda s}{na}$$

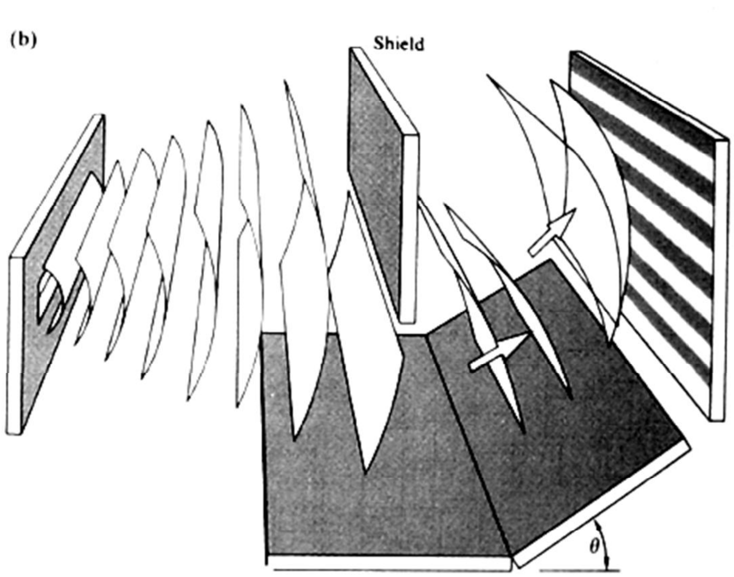
Fringes are more closely spaced under water.

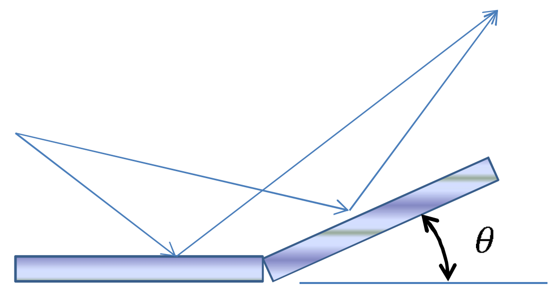
Optical path length is now $na \sin\theta$

(This gives us back the $a \sin\theta$ when $n = 1$)

Other Double-Slit Experiments

- Fresnel's double-mirror:

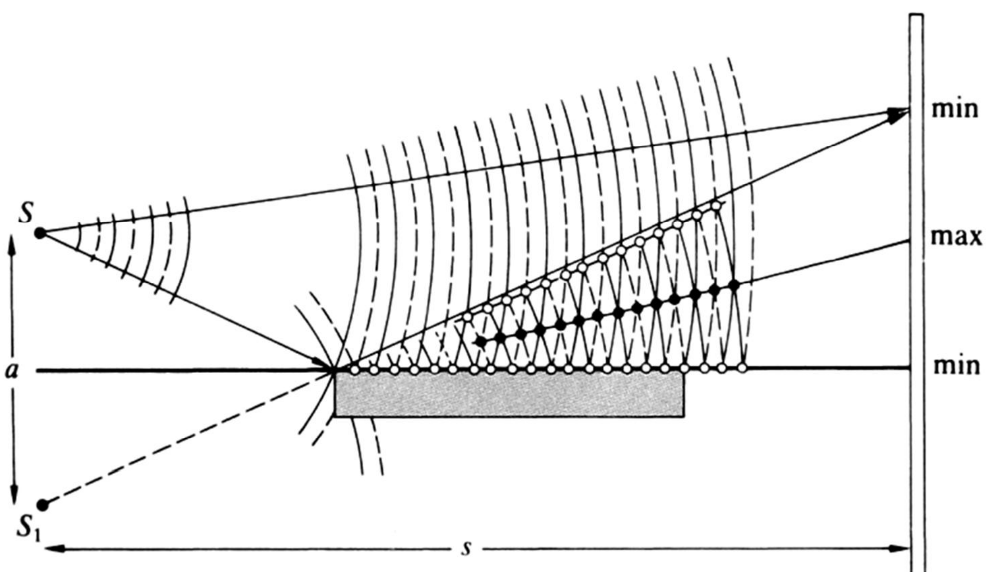




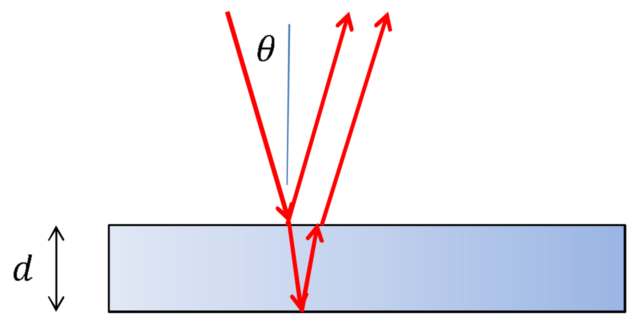
Mainly a problem of sorting out the geometry…

Lloyd's Mirror Interferometer

- Same geometry as a double-slit experiment except one light source is an image of the other one.
- But be careful! There is a phase shift of π for the reflected light. Destructive interference at $y = 0$.



Interference from Thin Films



- Phase difference for normal incidence:

$$\delta = 2\pi\left(\frac{2nd}{\lambda_0} + \frac{1}{2}\right)$$

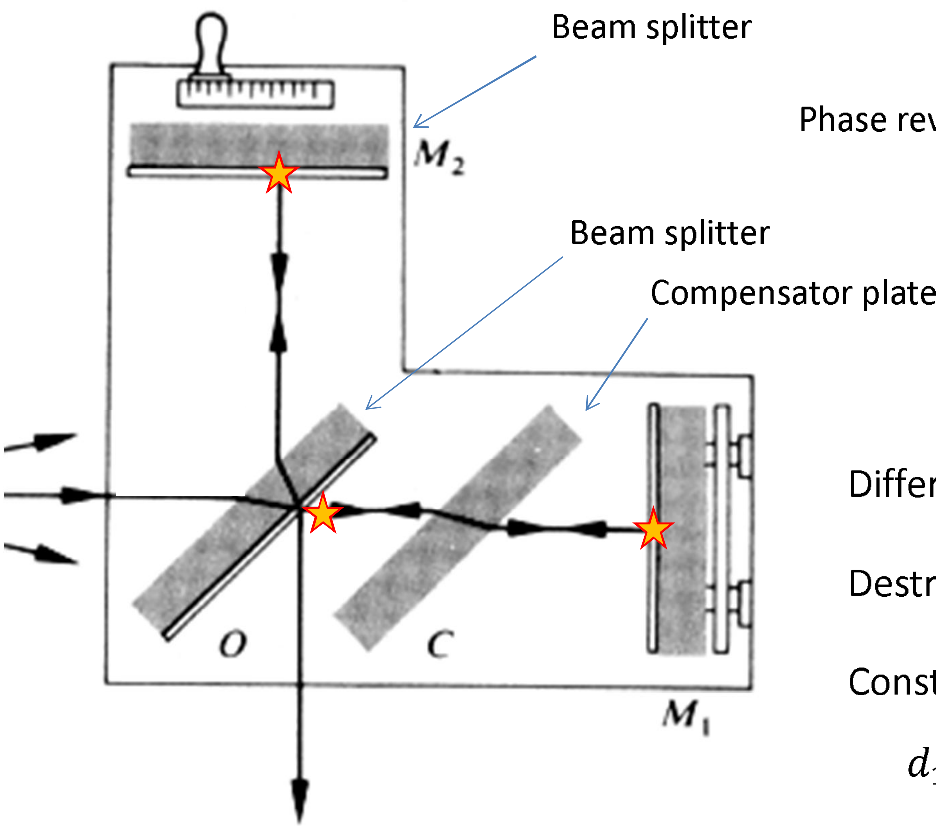
- Phase difference when angle of incidence is θ:

$$\delta = 2\pi\left(\frac{2nd}{\lambda_0 \cos\theta} + \frac{1}{2}\right)$$

- For monochromatic light, bright fringes have $\delta = 2\pi m$ and are located at

$$\cos\theta = \frac{nd}{\pi\lambda_0\left(m - \frac{1}{2}\right)}$$

Michelson Interferometer



Phase reversal: ★

Difference in path length:

$$d_1 - d_2$$

Destructive interference:

$$d_1 - d_2 = m\lambda$$

Constructive interference:

$$d_1 - d_2 = \left(m + \frac{1}{2}\right)\lambda$$

Michelson Interferometer

- Light is split into two separate paths, reflected off mirrors, then combined again.
- Be careful to count the phase reversals on reflection! (there is an odd number of them)
- Remember that the light travels in both directions! (be sure to double the distance to get the path length)
- Constructive interference occurs when

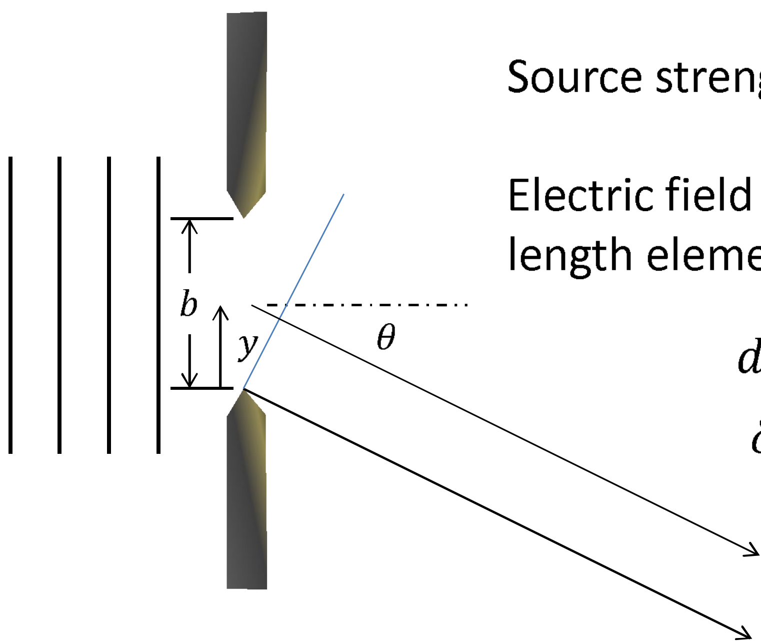
$$d_1 - d_2 = \left(m + \frac{1}{2}\right)\lambda$$

- Inserting something in one path changes the optical path length and shifts the interference fringes.
  - Counting the number of fringes that shift determines the change in optical path length precisely.

Fraunhofer Diffraction

- Far-field approximation: plane waves.
- Interference: sum over discrete point sources.
- Diffraction: integrate over continuous source of light where the phase depends on position.
- Fraunhofer diffraction: linear relation between phase and position.

Single-Slit Fraunhofer Diffraction



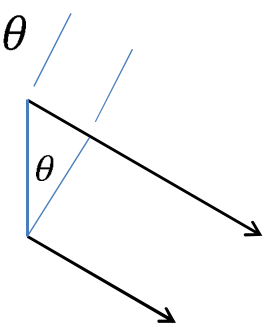
Light with intensity I_0 impinges on a slit with width b

Source strength per unit length: $\mathcal{E}_L$

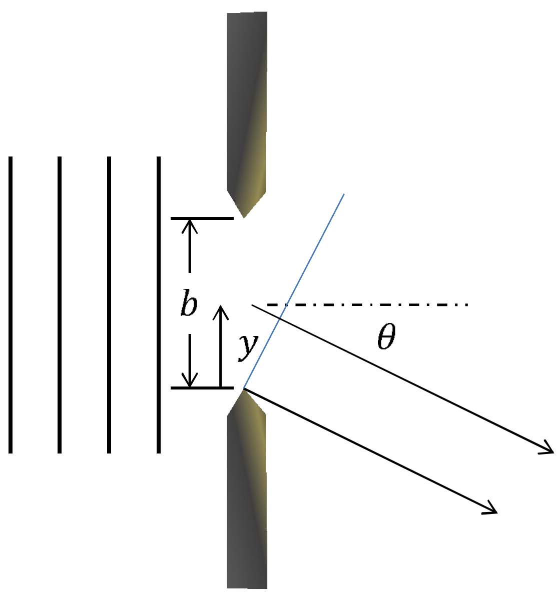
Electric field at a distance R due to the length element dy:

$$dE = \frac{\mathcal{E}_L e^{i\delta(y)} dy}{R}$$

$$\delta(y) = ky \sin\theta$$



Single-Slit Fraunhofer Diffraction



$$E = \frac{\mathcal{E}_L b}{R}\frac{\sin\beta}{\beta}$$

where

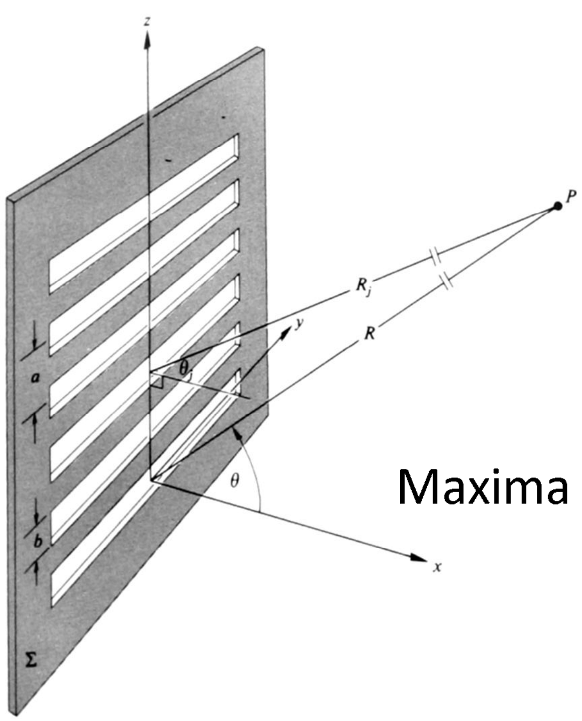
$$\beta = \frac{1}{2}kb\sin\theta$$

The intensity of the light will be

$$I(\theta) = I(0)\left(\frac{\sin\beta}{\beta}\right)^2$$
$$= I(0)\,\mathrm{sinc}^2\,\beta$$

Minima occur when $\beta = \pm\pi, \pm 2\pi, \ldots$

Multiple-Slit Fraunhofer Diffraction



- Main result:
  - when there are N slits:

$$I(\theta) = I(0)\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin N\alpha}{\alpha}\right)^2$$

$$\beta = \frac{1}{2}kb\sin\theta$$

$$\alpha = \frac{1}{2}ka\sin\theta$$

Maxima occur when $\alpha = \frac{1}{2}ka\sin\theta = m\pi$

$$a\sin\theta = m\lambda$$

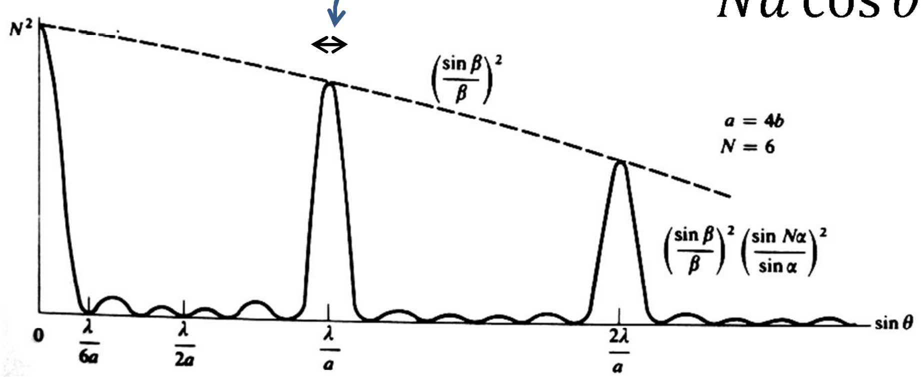
Multiple-Slit Fraunhofer Diffraction

- Light with a given wavelength is concentrated at specific angles:

$$\sin \theta_m = \frac{m\lambda}{a}$$

- Width of each line:

$$(\Delta\theta)_{min} = \frac{\lambda}{Na\cos\theta}$$



Diffraction Gratings

- Main application: measuring wavelengths of light
- Desirable features: high resolution
  - Should be able to distinguish two closely spaced spectral lines
- Chromatic resolving power:

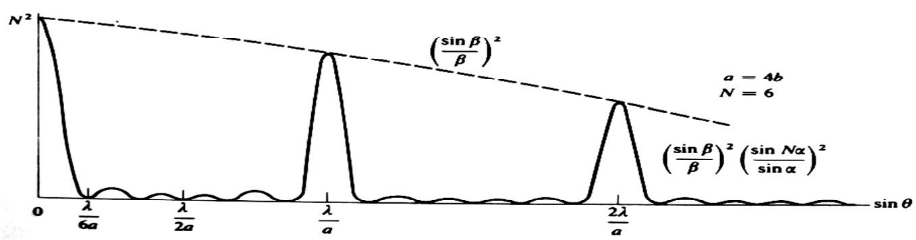
$$\mathcal{R} \equiv \frac{\lambda}{(\Delta\lambda)_{min}}$$

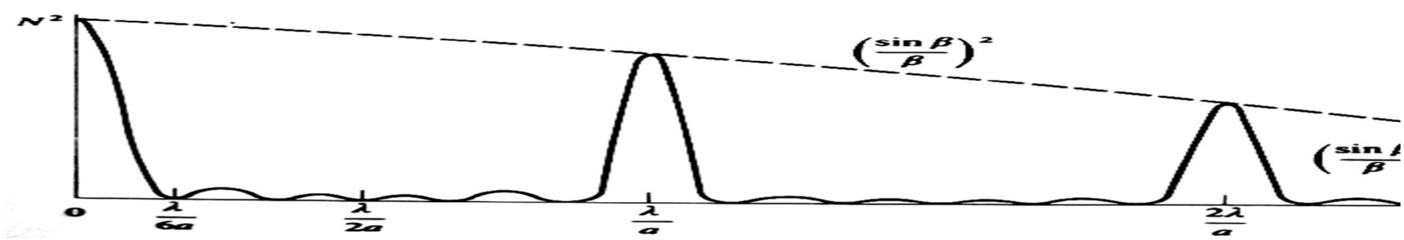
$$(\Delta\lambda)_{min} = \frac{\lambda}{Nm}$$

Diffraction Gratings

- Free spectral range: range of wavelengths for which short wavelengths in one order do not overlap with long wavelengths of the next order.

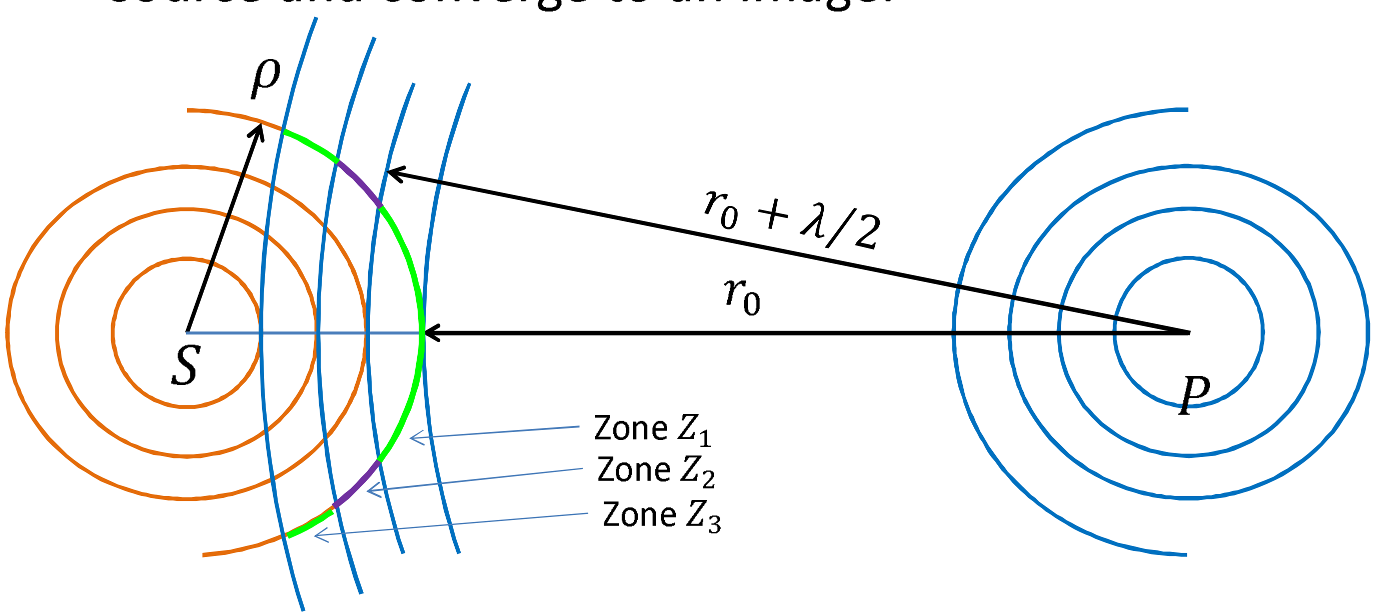
$$\frac{\lambda}{m} \equiv (\Delta\lambda)_{fsr}$$



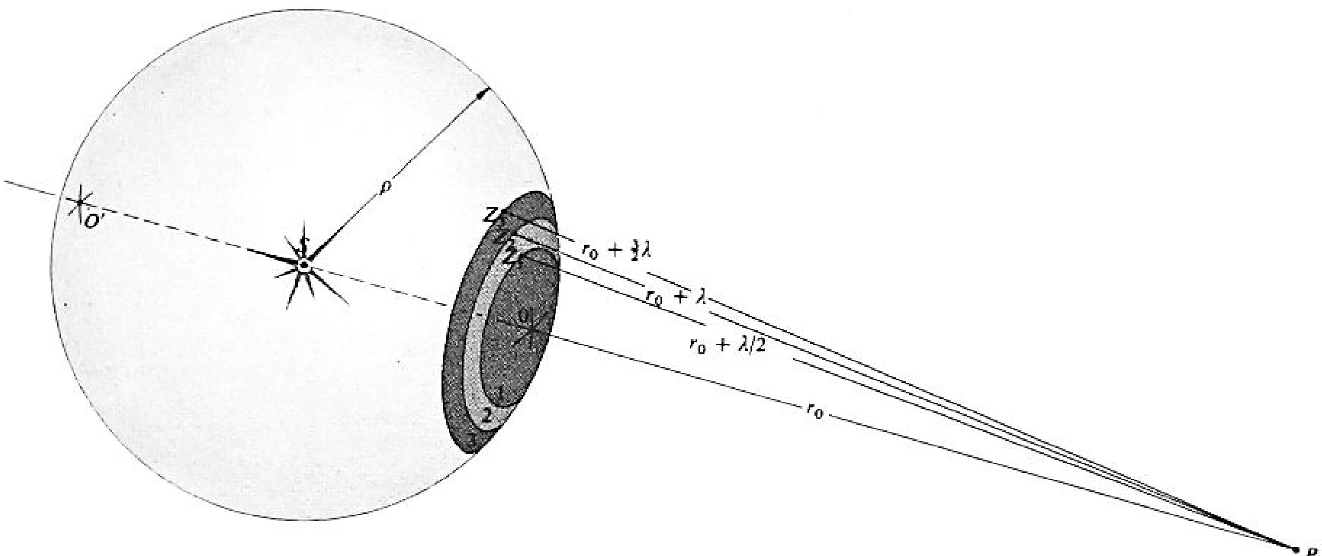


Fresnel Diffraction

- Main idea: how to think about different phases of spherical wave fronts that diverge from a point source and converge to an image.



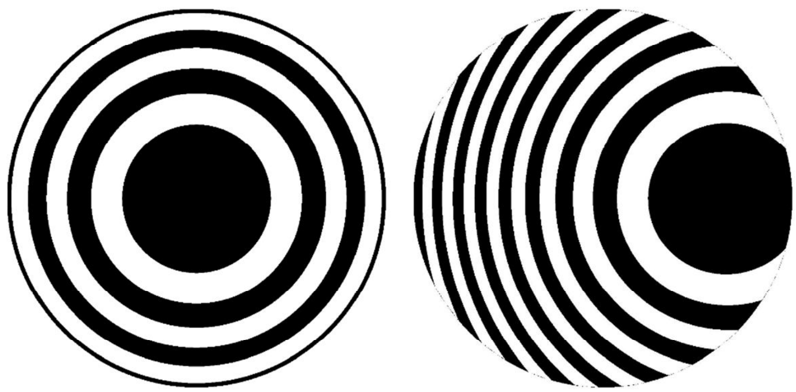
Fresnel Diffraction

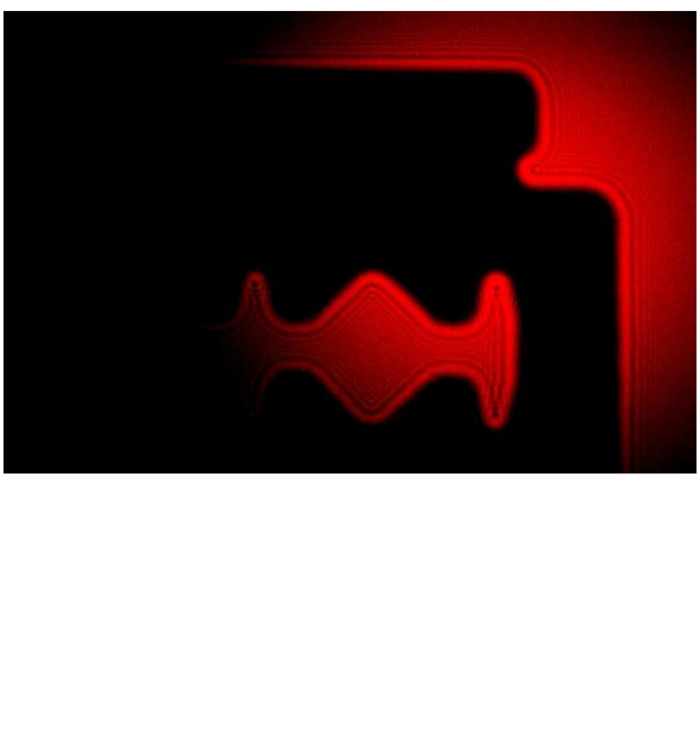


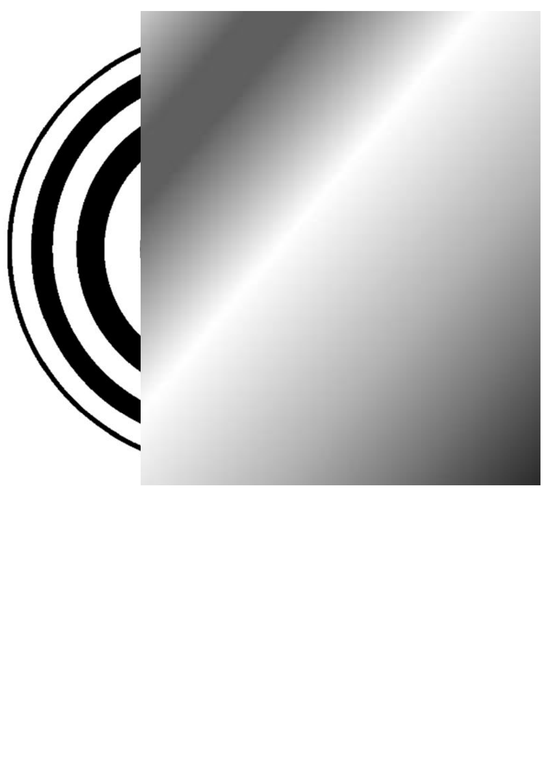
Properties of Fresnel “zones”:

- Equal light intensity arriving at image point from each zone.
- Phases of light in each zone alternate.
- Net constructive/destructive interference depends on how the zones are obscured by an aperture or obstruction.

Fresnel Diffraction

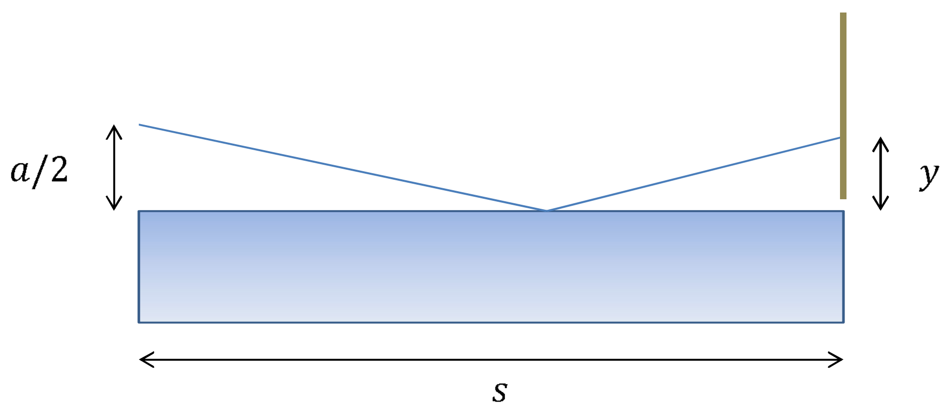






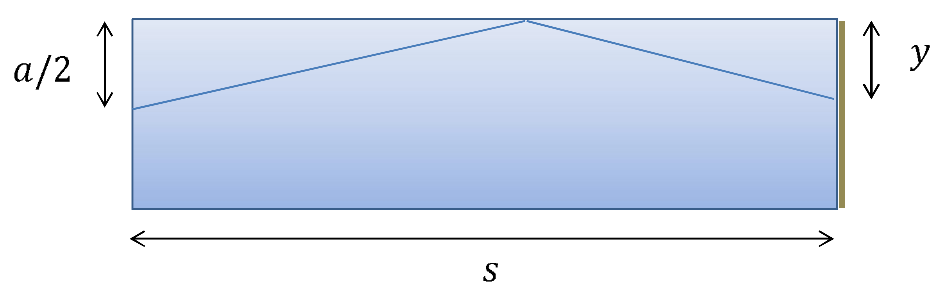
Typical Exam Questions

- A light source is located at a height $a/2$ above a container of water which acts like the mirror in Lloyd's interferometer. If a screen is located at a distance s from the light source, at what height, y, would the bright fringes appear?



Typical Exam Questions

- A light source is located at a height $a/2$ ***below*** a container of water (n>1) which acts like the mirror in Lloyd's interferometer. If a screen is located at a distance s from the light source, at what height, y, would the bright fringes appear?



Typical Exam Question

- A Michelson interferometer is used to measure the refractive index of (something) that is 10 cm long, placed in one arm of the interferometer. How many fringes will shift when the sample is heated, cooled, squished, stretched or otherwise abused by a given amount?
  - Calculate the difference in optical path length
  - Need to know the functional dependence of n
  - Need to know the wavelength of the light

Typical Exam Questions

- Answer one, or the other, but not both…

(a) Describe two types of optical aberration and techniques that can be used to minimize them.

(b) Compare and contrast the assumptions and main features of Fraunhofer and Fresnel diffraction. Under what circumstances is Fraunhofer diffraction a special case of Fresnel diffraction?