The Biot - Savart Law

Steady Currents

Stationary charges produce electric fields that are constant in time; hence the term electrostatics. Steady currents produce magnetic fields that are constant in time; the theory of steady currents is called magnetostatics

Stationary charges $\Rightarrow$ Constant electric fields : electrostatics

Steady currents $\Rightarrow$ Constant magnetic fields: magnetostatics.

By steady currents means a continuous flow that has been going on forever, without change and without charge piling up anywhere. Formally, electro/magnetostatics is the regime

$$\frac{\partial \rho}{\partial t} = 0, \quad \frac{\partial J}{\partial t} = 0$$

at all places and all times.

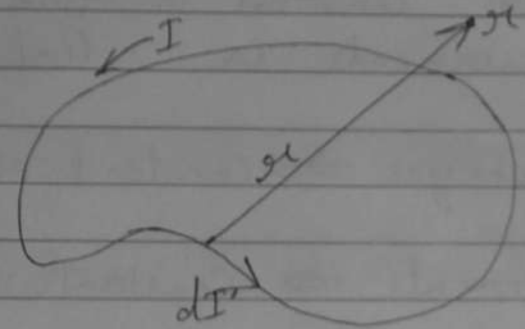
The Magnetic Field Of a Steady Current

The magnetic field of a steady line current is given by the Biot - Savart Law;

$$B(r) = \frac{\mu_0}{4\pi} \int \frac{I \times \hat{r}}{r^2} \, dl' = \frac{\mu_0}{4\pi} I \int \frac{dI' \times \hat{r}}{r^2}$$

The integration is along the current path, in the direction of the flow; dI' is an element of length along the wire, and r, as always, is the vector from the source to the point r as shown in fig. The constant μ_0 is called the Permeability of free space:

$$\mu_0 = 4\pi \times 10^{-7}\, N/A^2$$

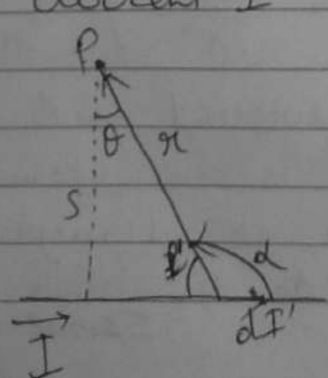


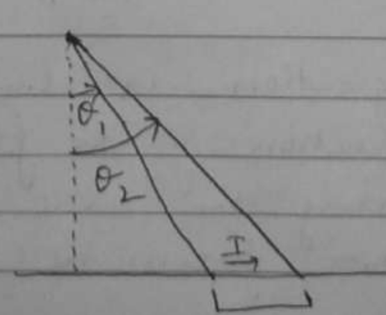
These units are such that B itself comes out in newtons per ampere-meter or teslas (T):

$$1\,T = 1\,N/(A.m).$$

As the starting point for magnetostatics, the Biot-Savart law plays a role analogous to Coulomb's law in electrostatics. Indeed, the $1/r^2$ dependence is common to both laws.

Que Find the magnetic field a distance s from a long straight wire carrying a steady current I





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Solution → In the diagram $(dI' \times \hat{r})$ points out of the page, and has the magnitude

$$dl' \sin\alpha = dl' \sin\theta$$

Also $l' = s\tan\theta$, so

$$dl' = \frac{s}{\cos^2\theta}\, d\theta$$

and $s = r\cos\theta$, so

$$\frac{1}{r^2} = \frac{\cos^2\theta}{s^2}$$

Thus

$$B = \frac{\mu_0 I}{4\pi}\int_{\theta_1}^{\theta_2}\left(\frac{\cos^2\theta}{s^2}\right)\left(\frac{s}{\cos^2\theta}\right)\cos\theta\, d\theta$$

$$= \frac{\mu_0 I}{4\pi s}\int_{\theta_1}^{\theta_2}\cos\theta\, d\theta = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1) \quad \rightarrow (1)$$

Equation 1 gives the field of any straight segment of wire, in terms of the initial and final angles θ_1 and θ_2 as in fig. Of course, a finite segment by itself could never support a steady current, but it might be a piece of some closed circuit, and equation 1 would then represent its contribution to the total field. In the case of an infinite wire, $\theta_1 = -\pi/2$ and $\theta_2 = \pi/2$, so we obtain

$$B = \frac{\mu_0 I}{2\pi s} \quad \text{—— (2)}$$

Notice that the field is inversly proportional to the distance from the wire — just like the electric field of an infinite line charge. In the region below the wire, B points into the page, and in general, it "circles around" the wire, in accordance with the right-hand rule;

$$B = \frac{\mu_0 I}{2\pi s} \hat{\phi}.$$

As an application, let's find the force of attraction between two long, parallel wire a distance d apart, carrying currents I_1 and I_2, The field at (2) due to (I) is

$$B = \frac{\mu_0 I_1}{2\pi d}$$

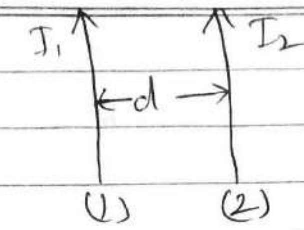
and it points into the page. The Lorentz Force Law predicts a force directed towards (I), of magnitude

$$F = I_2 \left(\frac{\mu_0 I_1}{2\pi d} \right) \int dl$$

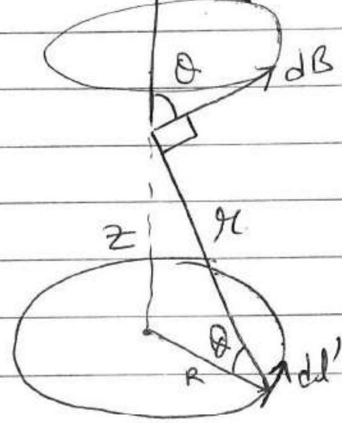
The total force, not surprisingly, is infinite, but the force per unit length is

$$f = \frac{\mu_0 I_1 I_2}{2\pi d}$$

If the currents are antiparallel (one up, one down), the force is repulsive — consistent again with the qualitative observations.



Que Find the magnetic field a distance z above the center of a circular loop of radius R, which carries a steady current I



Solution →
The field dB attributable to the segment dI' points as shown. As we integrate dI' around the loop, dB sweeps out a cone. The horizontal components cancel, and the vertical components combine, to give

$$B(z) = \frac{\mu_0}{4\pi} I \int \frac{dl'}{r^2} \cos\theta$$

(Notice that dI' and r are perpendicular, in this case; the factor of cos θ projects out the

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vertical component.) Now, $\cos\theta$ and $\mathscr{r}^2$ are constants, and $\int dl'$ is simply the circumference $2\pi R$, so

$$B(z) = \frac{\mu_0 I}{4\pi}\left(\frac{\cos\theta}{\mathscr{r}^2}\right)2\pi R = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}$$

Reference:- Introduction to Electrodynamics:- David J. Griffiths