$\cancel{f''(0)=}$

Now $f''(3) = 12 \times 9 - 24 \times 3 = 108 - 72$
$= 36 > 0$

$\Rightarrow x_0 = 3$ is ~~pt of~~ pt of local minimum

Local minimum value

$f(3) = (3)^4 - 4 \times 27 + 10 = 81 - 108 + 10$
$= 91 - 108 = -17$

Local minimum $= -17$

$\because f''(0) = 0 \Rightarrow$ test fail

so further

$f'''(x) = 24x - 24$ → 3 times diff which is odd.

$f'''(0) = -24 \neq 0$

$\Rightarrow$ at $x = 0$ neither max nor minimum.

→ using this result

let $f'(x_0) = f''(x_0) \cdots f^{n-1}(x_0) = 0$ but $f^n(x_0) \neq 0$ then

(1) If n is odd. $\Rightarrow f$ has neither local maxima nor local maxi at $x = x_0$

(2) If n is even then f has local maxima if $f^n(x_0) < 0$ & has local ~~max~~ min if $f^n(x_0) > 0$

Ans $y = x^4 - 4x^3 + 10$ sketch the graph.

Step 1 Find the domain of $x^4 - 4x^3 + 10$

$\therefore$ $y = f(x) = x^4 - 4x^3 + 10$ is poly & we know that poly defined $\forall x \in R$

$\Rightarrow$ domain of $f(x)$ is $R = (-\infty, \infty)$

Step 2 find $f'(x)$ & $f''(x)$

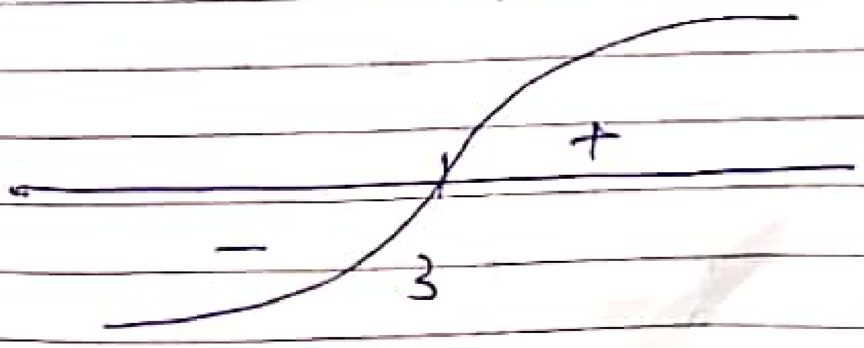
$f'(x) = 4x^3 - 12x^2$, $f''(x) = 12x^2 - 24x$

Step 3 critical point

$f'(x) = 0$ i.e $x = 0, 3$

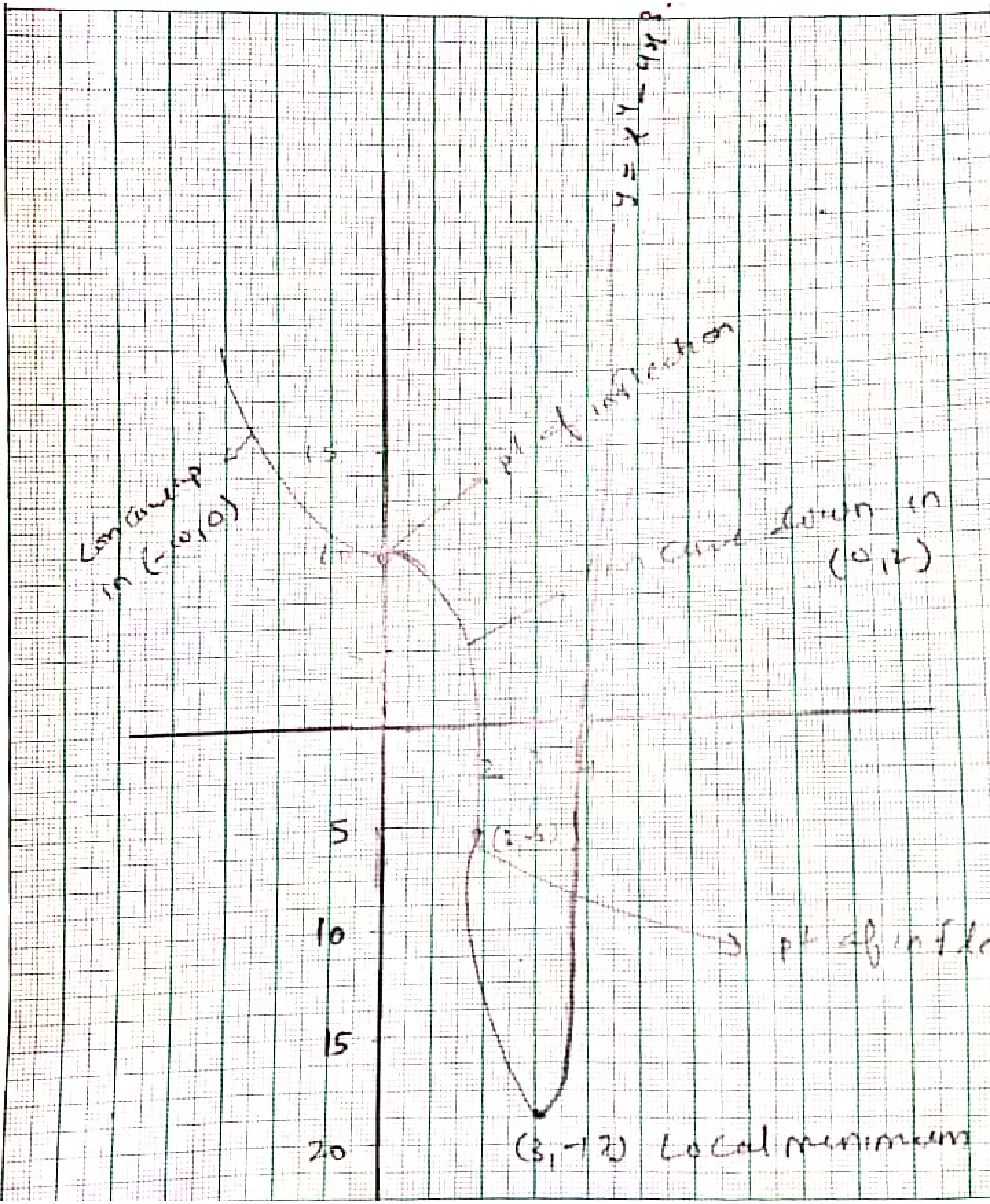
Step 4 Find the interval when $f(x)$ is decr or increasing

$\because$ $f'(x) = 4x^2(x-3)$



$f'(x) > 0 \quad \forall x \in (3, \infty)$

& $f'(x) < 0 \quad \forall x \in (-\infty, 3)$



y = x^4 − 4x^3
Concave up in (−∞, 0)
pt of inflection
concave down in (0, 2)
pt of inflection
5
10
15
20
(3, −27) Local minimum

Step 8 find vertical & horizontal asymptote

→ vertical asymptote a ~~line~~ line $x = 0$ is vertical asymptote of the graph $f(x)$ if either one sided limit

$\lim_{x \to a^-} f(x)$ or $\lim_{x \to a^+} f(x)$ is infinite

→ horizontal asymptote a line $y = b$ is a horizontal asymptote of graph f if $\lim_{x \to +\infty} f(x) = b$ or $\lim_{x \to -\infty} f(x) = b$

∵ $f(x) = x^4 - 4x^3 + 10$.

∄ $x = 0$ s.t. $\lim_{x \to a^+} f(x)$ or $\lim_{x \to a^-} f(x)$ is infinite

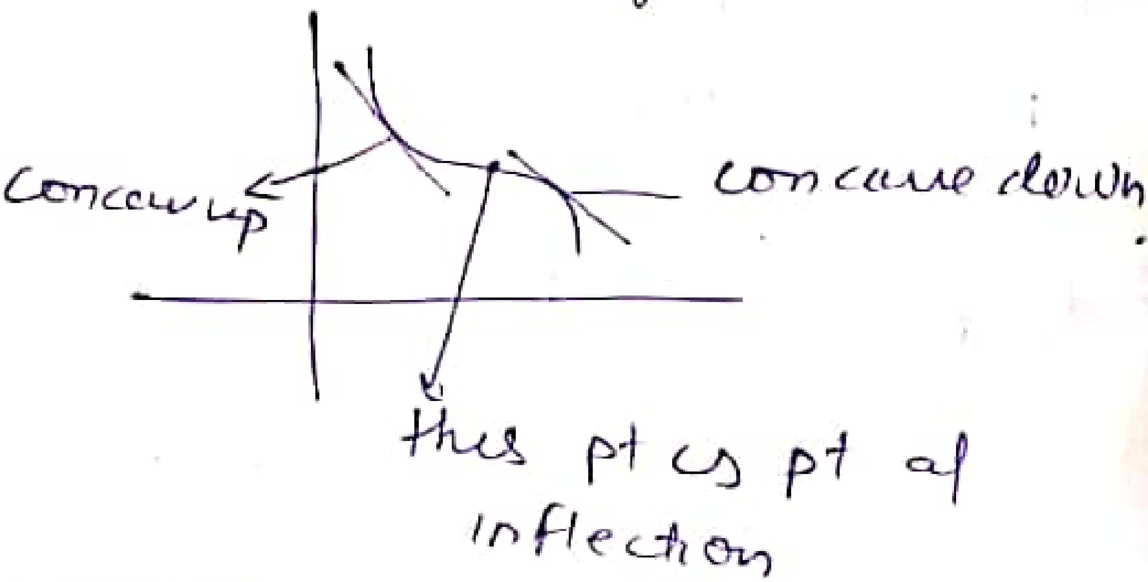
⇒ no vertical asymptote

∴ $\lim_{x \to \infty} f(x) = \infty \neq$ finite ⇒ no horizontal asymptote at $x = 3 \Rightarrow f'(3) = 0$

$\Rightarrow f(x)$ is increasing on $(3, \infty)$ & dec on $(-\infty, 3)$

Step 5 find pt of inflection.

Def^n A pt on the graph of $y = f(x)$ at which f changes concavity from up to down or down to up is called a pt of ~~inflexi~~ Inflection



now

$f''(x) = 12x^2 - 24x = 0 \Rightarrow 12x(x-2) = 0$

$x = 0, 2$

$f''(x) > 0$ on $(-\infty, 0) \cup (2, \infty)$

$\Rightarrow f$ is concave up on $(-\infty, 0) \cup (2, \infty)$

Procedure for Graphing the fn $y = f(x)$

Step (1) Find Domain of $f(x)$
2 find $f'(x)$ and $f''(x)$
3 find critical points. i.e find the point at which $f'(x) = 0$ or f' is not defined
4 Find the interval where fn is increase or decrease.

5 Identify any points of inflection.

6 find interval on ~~~~ which the fn is concave up or concave down.

7 find extreme values. use either the first derivative test or the second derivative test to determine the extreme value.

8 Identify any asymptotes. vertical or horizontal asymptotes.

9 find the intercepts with the co-ordinate axes y-intercept is obtained by setting $x=0$, and x-intercepts are the real roots (if any) of the eqn $f(x) = 0$

10 use the results of the above steps to sketch the graph.

$\therefore$ $f''(x) < 0$ on $(0,2)$

$\Rightarrow$ $f(x)$ is concave down on $(0,2)$

concave up : If $f'' > 0$ $\forall x \in I$ then fn is concave up on I

$\rightarrow$ If $f'' < 0$ $\forall x \in I$ then fn is concave down on I

$\therefore$ $f''(x)$ changes its sign at 2 & 0

$\Rightarrow$ $f(x)$ changes its concavity at pt 0 & 2

$\Rightarrow$ $x = 0, 2$ are the pt of inflection

& $f(0) = 10$ $(0,10)$ & $f(2) = 16 - 32 + 10 = -6$

i.e $(0,10)$ & $(2,-6)$ pt of inflection graph.

Step 6 find the interval where fn is concave up or concave down

$\because$ $f''(x) > 0$ on $(-\infty, 0) \cup (2, \infty)$

$\Rightarrow$ f is concave up on $(-\infty, 0) \cup (2, \infty)$

$\because$ $f''(x) < 0$ on $(0,2)$

$\Rightarrow$ f is concave down on $(0,2)$

Step 7 find extremum value

critical pt are 0, 3