Course: (CBCS) B.Sc. (H)-Physics [Section-A]

(32221402) Elements of Modern Physics

Part & Semester – II & IV

Lecture-3

Dear Students

Hope all of you are well and taking all the necessary precautions in this difficult time.

In our last class, we have discussed numerical problems on Basic Nuclear Properties, Binding Energy and Liquid Drop Model: Semi-empirical Mass Formula. In continuation of my last lecture, I am going to discuss about the following topics:

1- Nature of nuclear force
2- **NZ graph (From "Concepts of Modern Physics (Sixth Edition)" by Arthur Beiser, Sixth Edition, Chapter-11, Topic-11.3)**
3- Radioactivity: Stability of the nucleus
4- Law of radioactive decay
5- Mean-life and half-life and related problems

In our next lecture, we will discuss about the α, β, γ decay in detail. Apart from this, all of you can contact me through Email, Whatsapp or Mobile for any quarry related to our course of Elements of Modern Physics.

Thanking you.

With Best Wishes

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(April 04, 2020, Friday)

- **Nuclear forces:**

According to Coulomb's law, the positively charged protons closely spaced within the nucleus should repel each other strongly and they should fly apart. It is therefore difficult to explain the stability of nucleus unless one assumes that nucleons are under the influence of some very strong attractive forces. The forces inside the nucleus binding neutron to neutrons, protons to protons and neutrons to protons are classified as strong interactions and are represented as n–n, p–p and n–p forces respectively.

Characteristics of nuclear forces:

(i) They are short range forces i.e. the forces between nucleons are attractive in nature when they are 0.5-25F apart and these forces are of short range having maximum value at about 2×10^{-15} m and falls off sharply with distance, becoming negligible beyond this range,

(ii) They are charge-independent i.e. They are charge-independent so that the nuclear force between a proton and neutron and are almost the same;

(iii) They are the strongest known forces in nature;

(iv) They get readily saturated by the surrounding nucleons i.e. a particular nucleon interacts with a limited number of nucleons around it and other surrounding ones remain unaffected. So, they become saturated over short distance.

(v) They are spin-dependent ie. the nuclear forces depend on the mutual orientation of spins of various nucleons and are different in parallel and antiparallel spins.

- **Radioactivity:**
- **Laws of radioactive decay (Disintegration):**

(i) Atoms of every radioactive substance are constantly breaking into fresh radioactive products with the emission of α, β and γ rays.

(ii) The rate of breaking is not affected by external factors (temperature, pressure, chemical combination etc.) but is based upon probability concept and depends entirely on the law of chance i.e., the no. of atoms breaking per second at any instant is proportional to the number present.
If there are N atoms of any substance at time 't' and a number dN breaks in time dt

Then Rate of breaking $R = \frac{-dN}{dt} \propto N \quad \Rightarrow R = \frac{-dN}{dt} = \lambda N$

λ = radioactive constant = ratio of the amount of the substance which disintegrates in a unit time to the amount of substance present

$$\Rightarrow \qquad \frac{dN}{N} = -\lambda dt \qquad \Rightarrow \log_e N = -\lambda t + c$$

Now when $t = 0$, $\quad N = N_0 \quad \Rightarrow \log_e^{N_0} = C \quad \Rightarrow \quad \log_e \frac{N}{N_0} = -\lambda t$

Now, $\qquad N = N_0 e^{-\lambda t} \qquad \Rightarrow \frac{dN}{dt} = -\lambda N_0 e^{-\lambda t}$

N_0 = number of atoms at $t = 0$ and N = number of atoms left after time t.

i.e., $\qquad N_0 - N \rightarrow$ Converted to daughter.

Growth of daughter $= N_0 - N = N_0 - N_0 e^{-\lambda t} = N_0\left(1 - e^{-\lambda t}\right)$

At time $t = \frac{1}{\lambda}$, the no. of atoms of radioactive substance left behind is given by $N = N_0 e^{-\lambda \times \frac{1}{\lambda}} = \frac{N_0}{e}$

Hence the radioactive constant is also defined as the reciprocal of the time during which the number of atoms of a radioactive substance falls to 1/e of its original value.

- **Average life:** The atoms of a radioactive substance are constantly disintegrating and then the life of every atom is different. The atoms which disintegrate earlier have a very short life and others which disintegrates the end have a long life.

$$\text{Avgerage life} = \frac{\text{The sum of the lives of all atoms}}{\text{Total number of atom}}$$

We have $\frac{dN}{dt} = -\lambda N \qquad \Rightarrow -dN = \lambda N dt = \lambda N_o e^{-\lambda t} dt$

Total life of $-dN$ atom $= -tdN$

Since the possible life of any one of the total no. of atoms varies from 0 to ∞, the total life of all N_0 atoms is given by $\int_0^{N_0} -tdN$.

Average life, $\qquad T_a = \frac{1}{N_0}\int_0^{N_0} -tdN = \frac{1}{N_0}\int_0^{\infty} \lambda N_0 e^{-\lambda t} tdt = \lambda \int_0^{\infty} e^{-\lambda t} tdt$

$$= \lambda\left[\frac{e^{-\lambda t}}{-\lambda}t - \int\frac{e^{-\lambda t}}{-\lambda}dt\right]_0^\infty = \lambda\left[\frac{e^{-\lambda t}}{-\lambda}t - \frac{e^{-\lambda t}}{\lambda^2}\right]_0^\infty = -\frac{1}{\lambda}\left[(\lambda t+1)e^{-\lambda t}\right]_0^\infty = \frac{1}{\lambda}$$

- **Half life:** We have $N = N_0 e^{-\lambda t}$

  At half life $\frac{N_0}{2} = N_0 e^{-\lambda T} \Rightarrow \frac{1}{2} = e^{-\lambda T} \Rightarrow 2 = e^{+\lambda T} \Rightarrow \lambda T = \log_e 2$

  $T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{0.6931}{\lambda}$

- **Relation between half life and average life:** $T_{1/2} = 0.693\,T_a$
- **Units of Radioactivity:**

  SI unit Bacqurrel.

  1 Bq = 1 disintegration per second

  1 curie = 3.70×10^{10} disintegration per sec.

  1 Rutherford = 10^6 disintegration per second.

- **Activity or strength:**

  The activity or strength A_t of a radioactive sample at any instant 't' is thus defined as the number of disintegrations occurring in the sample in unit time at 't', that is,

  Activity, $A_t = \left|\frac{dN_t}{dt}\right| = \lambda N_t = \frac{0.693}{T}N_t$

  The activity per unit mass of a sample is called its specific activity.

  Differentiating the decay equation, $N_i = N_0 e^{-\lambda t}$, with respect to time 't'.

  $$\frac{dN_t}{dt} = -\lambda N_0 e^{-\lambda t}$$

  When t = 0, $\left(\frac{dN_t}{dt}\right)_0 = -\lambda N_0$. Hence from the relation (i) above

  $$\frac{dN_t}{dt} = \left(\frac{dN_t}{dt}\right)_0 e^{-\lambda t}$$

  Or, $A_t = A_0 e^{-\lambda t}$

  where, $A_t = dN_t/dt$ and $A_0 = (dN_t/dt)_0$

  A_t is called the activity or the strength of the sample and is proportional to the rate of disintegration.

- **Theory of Successive Disintegration:**

  Consider the equilibrium which is set up when radioactive body A disintegrate into a radioactive body B which disintegrated into a radioactive C. Let N_1 and N_2 be the no. of atom of A and B respectively present in the mixture after a time t and N_o the no. of atoms of A at start (t = 0). If λ_1 is the disintegration constant, then for A. $\Rightarrow N_1 = N_0 e^{-\lambda_1 t}$

  Every time an atom of A (called the parent) disappears, an atom B (called the daughter) is produced. Rate of formation of the daughter $B = \lambda_1 N_1$

Rate at which B decays $= \lambda_2 N_2$

Hence the Net rate at which B is produced $= \frac{dN_2}{dt} = (\lambda_1 N_1 - \lambda_2 N_2) = \lambda_1 N_0 e^{-\lambda_1 t} - \lambda_2 N_2$

$$\Rightarrow \quad \frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_0 e^{-\lambda_1 t}$$

Multiply both sides by $e^{\lambda_2 t}$

$$\Rightarrow \quad \frac{dN_2}{dt} e^{\lambda_2 t} + \lambda_2 N_2 e^{\lambda_2 t} = \lambda_1 N_0 e^{(\lambda_2 - \lambda_1)t} \Rightarrow \frac{d}{dt}\left[N_2 e^{\lambda_2 t}\right] = \lambda_1 N e^{(\lambda_2 - \lambda_1)t}$$

$$\Rightarrow \quad N_2 e^{\lambda_2 t} = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 e^{(\lambda_2 - \lambda_1)t} + c$$

When $t = 0$, $N_2 = 0 \Rightarrow c = -\frac{\lambda_1}{\lambda_2 - \lambda_1} N_o$

$$\Rightarrow \quad N_2 e^{\lambda_2 t} = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_o \left[e^{(\lambda_2 - \lambda_1)t} - 1\right] \Rightarrow N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 \left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right].$$

- **Secular (or permanent) Equilibrium:**

  If A, the parent is very long lived i.e., half life T_1 of A >> Half life T_2 of B, then $\lambda_1 << \lambda_2$. In such cases λ_1 can be neglected as compared to λ_2 and $e^{-\lambda_2 t}$ can be neglected as compared to $e^{-\lambda_1 t}$ provided t is very large. Thus above equation becomes $N_2 = N_o \frac{\lambda_1}{\lambda_2} e^{-\lambda_1 t}$

  If $T_{1/2}$ of A very large then $e^{-\lambda_1 t} \approx 1$ $\quad \Rightarrow N_0 = N_1$ or $N_2 = N_1 \frac{\lambda_1}{\lambda_2}$ $\quad \Rightarrow \lambda_1 N_1 = \lambda_2 N_2$

- **Transient Equilibrium:**

  If $T_{1/2}$ of A is not very large as compared to the time during which we make observation, then $e^{-\lambda_1 t} \neq 1$ and λ_1 can't be neglected as compared to λ_2

  Thus, $\quad N_2 = N_o \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_1 t} = \frac{N_1 \lambda_1}{\lambda_2 - \lambda_1} \left(\because N_1 = N_o e^{-\lambda_1 t}\right) \Rightarrow \frac{N_2}{N_1} = \frac{\lambda_1}{\lambda_2 - \lambda_1}$

- **Radioactive displacement law:** During a radioactive disintegration the nucleus which undergoes disintegration is called parent-nucleus and that which remains after the disintegration is called a daughter nucleus

α**-decay:** When a radioactive nucleus disintegrates by emitting an α-particle, the atomic number decreases by two and mass number decreases by four. It can be represented as

$${}_Z X^A \rightarrow {}_{Z-2} Y^{A-4} + {}_2 He^4$$

Example: Radium (${}_{88}Ra^{226}$) is converted to radon (${}_{86}Rn^{222}$) due to α-decay as

$${}_{88}Ra^{226} \rightarrow {}_{86}Rn^{222} + {}_2 He^4$$

β**-decay:** When a radioactive nucleus disintegrates by emitting a β-particles, the atomic number increases by one and the mass number remains the same. It can be represented as

$${}_Z X^A \rightarrow {}_{Z+1} Y^A + {}_{-1} e^0$$

Example: Thorium ($_{90}Th^{234}$) is converted to protoactinium ($_{91}Pa^{234}$)

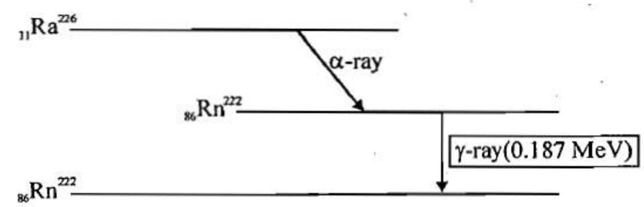
$$_{90}Th^{234} \rightarrow {}_{91}Pa^{234} + {}_{-1}e^{0}$$

At a time, either α or β-particles is emitted. Both α and β particles are not emitted during a single decay.

γ-ray: When a radioactive nucleus emits γ-ray, only the energy level of the nucleus changes and the atomic number and mass number remain the same.

During α or β decay, the daughter nucleus is mostly in the excited state. It comes to ground state with the emission of γ-rays.

Example: During the radioactive disintegration of radium ($_{88}Ra^{226}$) into Radon ($_{86}Rn^{222}$), gamma ray of energy 0.187 MeV is emitted, when radon returns from the excited state to the ground state as shown below:



Solved Examples

Example-1: A radioactive sample has its half-life equal to 60 days. Calculate its (i) disintegration constant, (ii) Its average life, (iii) the time required for 2/3 of the original number of atoms to disintegrate and (iv) the time taken for 1/4 of the original number of atoms to remain unchanged.

Soln. (i) Since $T_{1/2} = 60$ days, $\lambda = 0.693/T_{1/2} = 0.693/60 = 0.01155 \text{ day}^{-1}$

(ii) Since $\lambda = 0.01155 \text{ day}^{-1}$, $\overline{T} = 1/\lambda = 1/0.01155 = 86.58$ days

(iii) Number to be disintegrated $= \frac{2}{3}N_0$. So, the number to remain unchanged $= \frac{1}{3}N_0$

Thus, $\quad N/N_0 = \frac{1}{3}$

Therefore, $\quad$ From the relation $N = N_0 e^{-\lambda t}$, we obtain $\frac{1}{3} = e^{-\lambda t} \Rightarrow \lambda t = \ell n\, 3$

$$\Rightarrow \quad t = \frac{\ell n\, 3}{\lambda} = \frac{2.3026 \times 0.4771}{0.01155} = 95.1 \text{ days}$$

(iv) Here $N/N_0 = \frac{1}{4}$. So, from the decay law, $\frac{1}{4} = e^{-\lambda t} \Rightarrow \lambda t = \ell n 4$

$$\Rightarrow \quad t = \frac{\ell n\, 4}{\lambda} = \frac{2.3026 \times 0.6021}{0.01155} = 120 \text{ days}$$

Example-2: (a) A radioactive substance disintegrates for a time equal to its average life. Calculate the fraction of the original substance disintegrated.

(b) The half-life of a radon is 3.82 days. What fraction of freshly prepared sample of radon will disintegrate in 10 days?

Soln. (a) Here $\overline{T} = t = \frac{1}{\lambda}$ Since, $\frac{N}{N_0} = e^{-\lambda t} = e^{-\lambda\overline{T}} = e^{-1} = 0.368$

Therefore, Fraction disintegrated = 1–0.368 = 0.632

(b) Here $T_{1/2} = 3.82$ days and t = 10 days. Therefore, we have $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{3.82} = 0.181\ \text{day}^{-1}$

Now, $\frac{N}{N_0} = e^{-\lambda t} = e^{-0.181\times10} = e^{-1.81} = \frac{3}{20}$

$\Rightarrow$ Fraction disintegrated $= 1 - \frac{3}{20} = \frac{17}{20}$

Example-3: The half-life of UX_1 is 24.1 days. How many days, after UX_1 has been isolated, will it take for 90% of it to change to UX_2?

Soln. Here $T_{1/2} = 24.1$ days $\Rightarrow$ $\lambda = 0.693/24.1 = 0.0287\ \text{day}^{-1}$

Amount disintegrated = 90/100 $\Rightarrow$ $N/N_0 = 10/100 = 1/10$

Since, $\frac{N}{N_0} = e^{-\lambda t}$, $\frac{1}{10} = e^{-0.0287t}$

$$\Rightarrow \quad t = \frac{\ell n\, 10}{0.0287} = \frac{2.3026\times1}{0.0287} = 80\ \text{days}$$

Example-4: The half-life of radioactive K-40 is 1.83×10^{8} years. Find the number of $\beta-$particles emitted per sec per g. of K-40, assuming $\lambda = 1.2\times10^{-17}\ s^{-1}$, Avogrado number = 6.02×10^{23}.

Soln. Let N_t = Number of atoms of K-40 in 1g at $t = 6.02\times100^{23}/40$

$$\Rightarrow \quad \left|\frac{dN_t}{dt}\right| = \lambda N_t = \frac{1.2\times10^{-17}\times6.02\times10^{23}}{40} = 1.8\times10^{5}$$

So, the number of particles emitted per g of K-40 is 1.8×10^{5}

Example-5: It is observed that 3.67×10^{10} $\alpha-$particles are emitted per g of Ra- 226. Calculate the half-life of Ra-226. Avogadro number = 6.023×10^{23}.

Soln. 1g of Ra-226 = $6.023\times10^{23}/226$ atoms of Ra-226. Of these 3.67×10^{10} disintegrate per sec. So, the decay constant.

$$\lambda = \frac{3.67\times10^{10}}{6.023\times10^{23}/226}\ s^{-1} \quad \Rightarrow \quad T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693\times6.023\times10^{23}}{3.67\times10^{10}\times226} s = 1595\ \text{years}$$

Example-6: A large amount of radioactive material of half-life 20 days got spread in a room making the level of radiation 40 times the permissible level of normal occupancy. After how many days would the room be safe for occupation.

Soln. Let after 't' days the room would be safe for occupation. So, in 't' days, the activity would drop down to 1/40 of its initial value.

$\Rightarrow \quad \dfrac{N}{N_0} = \dfrac{1}{40}$. From the relation $\dfrac{N}{N_0} = e^{-\lambda t}, \dfrac{1}{40} = e^{-(0.693/20)t}$

$\left(\because T_{1/2} = 40 \text{ days}, \ \lambda = (0.693/20)\text{day}^{-1}\right) \quad \Rightarrow \ell n\, 40 = \dfrac{0.693}{20} t$

$\Rightarrow \quad t = \dfrac{2.303 \times 20}{0.693} \log_{10} 40 = \dfrac{2.303 \times 20 \times 1.6021}{0.693} = 106.4$

Therefore, room would be safe for occupation after 107 days.

Example-7: Calculate the amount of Ra-226 in secular equilibrium with 1 kg of pure U-238, given the half-lives of Ra-226 and U-238 as 1620 years and 4.5×10^9 years respectively.

Soln. Let x in g be the required amount of Ra-226 in secular equilibrium with 1 kg of pure U-238. Now:

$$1 \text{ kg of pure U-238} = \frac{6.02 \times 10^{23}}{238} \times 10^3 \text{ atoms of U}$$

$$x \text{ g of Ra-226} = \frac{6.02 \times 10^{23}}{226} \times x \text{ atoms of Ra}$$

The condition of secular equilibrium gives, $\dfrac{N_U}{(T_{1/2})_U} = \dfrac{N_{R_a}}{(T_{1/2})_{R_a}}$

$$\Rightarrow \quad \frac{6.02 \times 10^{23} \times 10^3}{238 \times 4.5 \times 10^9} = \frac{6.02 \times 10^{23} \times x}{226 \times 1620}$$

$$\Rightarrow \quad x = \frac{226 \times 1620 \times 10^3}{238 \times 4.5 \times 10^9} = 34.18 \times 10^{-5} \text{ g} = 0.34 \text{ mg}$$

Example-8: The Half life of ${}_{92}U^{238}$ is 4.51×10^9 yrs. What %age of ${}_{92}U^{238}$ that existed 10^{10} years ago still survives.

Soln. $\lambda = \dfrac{0.693}{t_{1/2}} = \dfrac{0.693}{4.5 \times 10^9}$

If N_0 = no. of atom of ${}_{92}U^{238}$ excited 10^{10} year ago $N = N_0$.

Now present, $N = N_0 e^{-\lambda t}$ where $t = 10^{10}$yrs $\Rightarrow \dfrac{N}{N_0} = e^{\lambda t}$ or $\log_e \dfrac{N_0}{N} = \lambda t$

$\Rightarrow \quad 2.3026 \log_{10} \dfrac{N_0}{N} = \lambda t = \dfrac{0.693 \times 10^{10}}{4.51 \times 10^9} \quad \Rightarrow \log_{10} \dfrac{N_0}{N} = \dfrac{0.693 \times 10}{2.3026 \times 4.51} = 0.6673.$

$\Rightarrow \quad \dfrac{N_0}{N} = \text{anti} \log 0.6673 = 4.648 \quad \Rightarrow \quad \dfrac{N}{N_0} = 0.215$

% of ${}_{92}U^{235}$ now present = $0.215 \times 100 = 21.5\%$

Example-9: The half life of a radioactive substance is 5 hr. What will be its one third life time?

Soln. $T_{1/2} = 5$ hrs. $T_{1/2} = \dfrac{0.693}{\lambda} \quad \Rightarrow \lambda = \dfrac{0.693}{5} = 0.1386$ per hour also $\dfrac{N}{N_0} = e^{-\lambda t}$.

In this case $\frac{N}{N_0} = \frac{1}{3} \Rightarrow \frac{1}{3} = e^{-\lambda t} \Rightarrow 3 = e^{\lambda t}$

Hence, $\log_e 3 = \lambda t$ or $t = \frac{2.3026 \log_{10} 3}{0.1386} = 7.93$ hrs

Example-10: The activity of certain radio nuclide decreases to 15% of its original value in 10 days. Find its half life.

Soln. Let N_0 be the original no. of nuclei and N left behind after 10 days. If λ is the radioactive constant, then

$$\Rightarrow \quad \frac{N}{N_0} = e^{-\lambda t} \Rightarrow \frac{15}{100} = e^{-\lambda 10}$$

$$\Rightarrow \log_e \frac{100}{15} = 10\lambda \Rightarrow \lambda = \frac{1}{10} \log_e \frac{100}{15} = \frac{1}{10} \times 2.3026 \log_{10} \frac{100}{15} = 0.1897$$

$$\Rightarrow \quad T_{1/2} = \frac{0.6931}{\lambda} = \frac{0.6931}{0.1897} = 3.65 \text{ day}$$

References:

1- Concepts of Modern Physics (Sixth Edition, TMH Pvt. Ltd.) by Arthur Beiser et.al.

2- Nuclear Physics (S. Chand Limited, 2008) by S. N. Ghoshal.

3- Nuclear Physics (Himalaya Publishing House, Mumbai) by D. C. Tayal.

4- Last year examination papers.

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To be cont…..