Thus, the aggregate excess dds of the 2 goods are:

$$Z_1(P_1, P_2) = a\frac{m_A}{P_1} + b\frac{m_B}{P_1} - \omega_A^1 - \omega_B^1$$

$$= a\frac{P_1\omega_A^1 + P_2\omega_A^2}{P_1} + b\frac{P_1\omega_B^1 + P_2\omega_B^2}{P_1} - \omega_A^1 - \omega_B^1 \quad \text{--- (9)}$$

and,

$$Z_2(P_1, P_2) = (1-a)\frac{m_A}{P_2} + (1-b)\frac{m_B}{P_2} - \omega_A^2 - \omega_B^2$$

$$= (1-a)\frac{P_1\omega_A^1 + P_2\omega_A^2}{P_2} + (1-b)\frac{P_1\omega_B^1 + P_2\omega_B^2}{P_2} - \omega_A^2 - \omega_B^2 \quad \text{--- (10)}$$

- Now, we should verify that these aggregate dd functions satisfy Walras' law.
- Let us take P_2 as the numeraire price, i.e, set $P_2 = 1$ then (9) and (10) becomes.

$$Z_1(P_1, 1) = a\frac{P_1\omega_A^1 + \omega_A^2}{P_1} + b\frac{P_1\omega_B^1 + \omega_B^2}{P_1} - \omega_A^1 - \omega_B^1 \quad \text{--- (11)}$$

and

$$Z_2(P_1, 1) = (1-a)[P_1\omega_A^1 + \omega_A^2] + (1-b)[P_1\omega_B^1 + \omega_B^2] - \omega_A^2 - \omega_B^2 \quad \text{--- (12)}$$

To get the equm (relative) price, set either of (11) or (12) equal to zero and solve for P_1

(*) According to Walras' law we should get the same equm price, no matter which equation we solve.

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- From equation (12)

$z_2(p_1, 1) = 0$

$\Rightarrow (1-a)[p_1 w_A^1 + w_A^2] + (1-b)[p_1 w_B^1 + w_B^2] - w_A^2 - w_B^2 = 0$

$\Rightarrow p_1 [(1-a) w_A^1 + (1-b) w_B^1] = w_A^2 + w_B^2 - (1-a) w_A^2 - (1-b) w_B^2$

$= a w_A^2 + b w_B^2$

$$\Rightarrow p_1^* = \frac{a w_A^2 + b w_B^2}{(1-a) w_A^1 + (1-b) w_B^1} \qquad (13)$$

(*) Try this with (11) also.

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Calculus Conditions for Pareto efficient allocations :

By definition, a Pareto efficient allocation makes each agent as well off as possible, given the utility of the other agent.

→ Let $U_B(x_B^1, x_B^2) = \bar{U}$ → a constant

Then, $\max\limits_{x_A^1, x_A^2, x_B^1, x_B^2} U_A(x_A^1, x_A^2)$

subject to $U_B(x_B^1, x_B^2) = \bar{U}$

$$x_A^1 + x_B^1 = \omega^1 \quad [\because = \omega_A^1 + \omega_B^1]$$

$$x_A^2 + x_B^2 = \omega^2 \quad [\because = \omega_A^2 + \omega_B^2]$$

The Lagrangean is

$$L = U_A(x_A^1, x_A^2) - \lambda[U_B(x_B^1, x_B^2) - \bar{U}] \qquad \text{— (1)}$$
$$- \mu_1[x_A^1 + x_B^1 - \omega^1] - \mu_2[x_A^2 + x_B^2 - \omega^2]$$

[λ → Lagrangean multiplier for the utility constraint and μ_1 & μ_2 → - - - - - resource contraints]

F.O.C.

$$\frac{\partial L}{\partial x_A^1} = \frac{\partial U_A}{\partial x_A^1} - \mu_1 = 0 \qquad \text{— (2)}$$

$$\frac{\partial L}{\partial x_A^2} = \frac{\partial U_A}{\partial x_A^2} - \mu_2 = 0 \qquad \text{— (3)}$$

$$\frac{\partial L}{\partial x_B^1} = -\lambda \frac{\partial U_B}{\partial x_B^1} - \mu_1 = 0 \qquad \text{— (4)}$$

$$\frac{\partial L}{\partial x_B^2} = -\lambda \frac{\partial U_B}{\partial x_B^2} - \mu_2 = 0 \qquad \text{— (5)}$$

$$\frac{\partial L}{\partial \lambda} = U_B(x_B^1, x_2^2) - \bar{U} = 0 \qquad \text{— (6)}$$

$$\frac{\partial L}{\partial \mu_1} = x_A^1 + x_B^1 - \omega^1 = 0 \qquad \text{— (7)}$$

$$\frac{\partial L}{\partial \mu_2} = x_A^2 + x_B^2 - \omega^2 = 0 \qquad \text{— (8)}$$

• Dividing (2) by (3) we get

$$MRS_A = \frac{\partial U_A / \partial x_A^1}{\partial U_A / \partial x_A^2} = \frac{\mu_1}{\mu_2} \quad \text{— (9)}$$

and, dividing (4) by (5) we get.

$$MRS_B = \frac{\partial U_B / \partial x_B^1}{\partial U_B / \partial x_B^2} = \frac{\mu_1}{\mu_2} \quad \text{— (10)}$$

from (9) and (10) $\longrightarrow$ $MRS_A = MRS_B$ at a Pareto efficient allocation

(*) While making optimal choices, both agents are maximising utility subject to her budget constraints, and both consumers face the same price for goods 1 & 2.

Then, $MRS_A = \frac{P_1}{P_2}$ (11) and $MRS_B = \frac{P_1}{P_2}$ — (12)

(*) Note, the similarity with the efficiency conditions. The Lagrange multiplier in the efficiency conditions μ_1 and μ_2 are just like the prices P_1 and P_2 in the consumer choice conditions. In fact the Lagrange multipliers in this kind of problem are sometimes known as shadow prices or efficiency prices.

• Every Pareto efficient allocation has to satisfy conditions like those in (9) and (10)

• Every competitive equm. has to satisfy conditions like those in (11) and (12)

(*) The conditions describing Pareto efficiency and the conditions describing individual maximisation in a mkt environment are virtually the same.

The Existence of Equilibrium:

- In the previous algebraic example we had specific equations for each consumer's dd func and we could explicitly solve for the equm prices.
- BUT, in general we do not have explicit algebraic formulas for each consumer's dds.

Q: How do we know that there is any set of prices such that dd = ss in every mkt ??

→ This is known as the question of the existence of a competitive equm

→ is important insofar as it serves as a "consistency check" for the various models that we have examined previously.

This is a primary Q.

→ i.e, there is no pt on working on the theory of a competitive equm if we are not even sure, whether it exists.

- Early economists noted that there are $(k-1)$ relative prices to be determined, and there are $(k-1)$ equm condns stating that dd should equal ss in each mkt.

→ Since the number of equations equalled the number of unknowns, they asserted that there would be a solution where all of the equations were satisfied.

→ these arguments were discovered to be fallacious quite soon.

• ie, merely counting the number of equations and unknowns is not sufficient to prove that an eqm soln will exist.

(*) BUT, mathematical tools are there to do that.

(*) The crucial assumption turns out to be that the aggregate excess dd fun is a continuous fun

→ small changes in prices → only small changes in the aggregate dd : (ie, not a big jump in qty dd)

(*) → under what conditions ?

→ 2 kinds of conditions

I) each individuals dd fun is continuous — this turns out to require that each consumer have convex preferences.

II) Even if consumers themselves have discontinuous dd behaviour, as long as all consumers are small relative to the size of the mkt, the aggregate dd fun will be continuous.

(*) → The latter condn is quite nice. After all, the assumption of competitive behaviour only makes sense when there are a lot of consumers who are small relative to the size of the mkt. → this is exactly the condn that we need in order to get the aggregate dd fun s to be continuous. And, continuity is just the ticket to ensure that a competitive eqm exists.

(*) Thus, the very assumptions that make the postulated behaviour reasonable will ensure that the eqm theory will have content.

Equilibrium and Efficiency:

- We have now analysed market trade in a pure exchange model. This gives us a specific model of trade that we can compare to the general model of trade that we discussed in the beginning.

Q ~~(?)~~ regarding the use of a competitive mkt
whether the mechanism can really exhaust all the gains from trade.
After we have traded to a competitive equm where dd = ss in every mkt, will there be any more trades that people will desire to carry out ??

— This is just another way to ask whether the mkt. equm is Pareto efficient: will the agents desire to make any more trades after they have traded at the competitive prices?

- The Ans is in the previous diagram (Fig 4)

→ the mkt equm is Pareto-efficient

Proof: In the Edgeworth box, the bundles that A prefers does not intersect with the set of bundles that B prefers.

At the same time at the mkt equm both agents preferred bundles must lie above their ~~budget~~ respective budget sets

→ the two sets of preferred allocations can't intersect

→ there are no allocations that both agents prefer to the equm allocation

→ the equm is Pareto-efficient

The Algebra of Efficiency :

- Suppose that, we have a mkt-equm that is not Pareto-efficient. We will show that this assumption leads to a logical contradiction.

- To see that the mkt equm is not Pareto efficient, means that there is some other feasible allocation $(y_A^1, y_A^2, y_B^1, y_B^2)$ such that

$$y_A^1 + y_B^1 = \omega_A^1 + \omega_B^1 \quad \text{——— (1)}$$

$$y_A^2 + y_B^2 = \omega_A^2 + \omega_B^2 \quad \text{——— (2)}$$

and,

$$(y_A^1, y_A^2) \succ_A (x_A^1, x_A^2) \quad \text{——— (3)}$$

$$(y_B^1, y_B^2) \succ_B (x_B^1, x_B^2) \quad \text{——— (4)}$$

(1) and (2) $\Rightarrow$ that the y-allocation is feasible

and (3) and (4) $\Rightarrow$ that the y-allocation is preferred by each agent to the x-allocation.

- BUT, by hypothesis, we have a mkt equm where each agent is purchasing the best bundle she can afford.
- Hence, if (y_A^1, y_A^2) is better than the bundle that A is choosing then it must cost more than A can afford, and similarly for B :

$$p_1 y_A^1 + p_2 y_A^2 > p_1 w_A^1 + p_2 w_A^2 \quad \text{——} (5)$$

$$p_1 y_B^1 + p_2 y_B^2 > p_1 w_B^1 + p_2 w_B^2 \quad \text{——} (6)$$

Adding (5) and (6)

$$p_1 (y_A^1 + y_B^1) + p_2 (y_A^2 + y_B^2) > p_1 (w_A^1 + w_B^1) + p_2 (w_A^2 + w_B^2) \quad \text{——} (7)$$

putting from (1) and (2) into (7)

$$p_1 (w_A^1 + w_B^1) + p_2 (w_A^2 + w_B^2) > p_1 (w_A^1 + w_B^1) + p_2 (w_A^2 + w_B^2) \quad \text{——} (8)$$

→ which is clearly a contradiction, since, the LHS and the RHS are the same.

reached by the assumption that the mkt equm was not Pareto efficient

↳ hence, this assumption must be wrong.

∴ All mkt equilibria are Pareto efficient — First theorem of Welfare Economics

↳ guarantees that a competitive mkt will exhaust all the gains from trade : an equm allocation achieved by a set of competitive mkts will necessarily be Pareto-efficient.

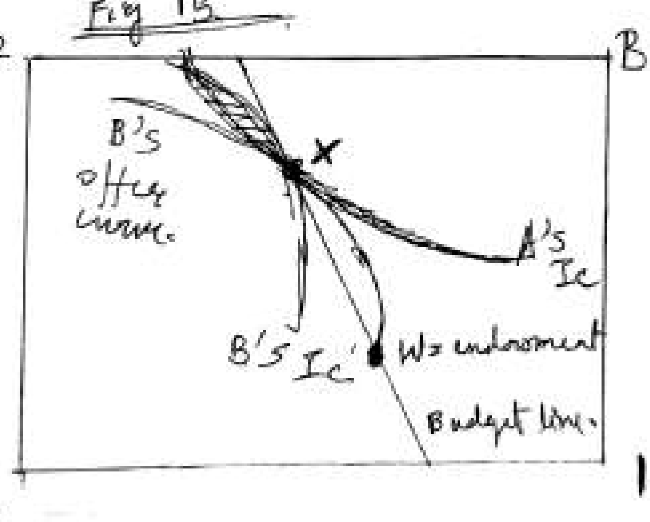
[may not have any other desirable properties but it will necessarily be efficient.]

(*) In particular, the First Welfare theorem says nothing about the distribution of economic benefits → i.e. the simple mkt. mechanism leads to an efficient allocation → BUT, Q. remains → how fair or just is that allocation ?

(4) EXAMPLE : Monopoly in the Edgeworth Box :

- In order to understand the First Welfare Theorem (FWT) better, it is useful to consider another resource allocation mechanism, that does not lead to efficient outcomes.
- Example → when one consumer attempts to behave as a monopolist
- Suppose, now there is no auctioneer and that instead, agent A is going to quote prices to agent B, and agent B will decide how much he wants to trade at the quoted prices
- Suppose further, that A knows B's dd curve and will attempt to choose the set of prices that makes A as well off as possible, given the dd behaviour of B.
- In order to examine the equm in this process — recall the definition of a consumer's price offer curve (PCC).
  → Then, B's offer curve represents the bundles that he will purchase at different prices : ie, it describes B's dd behaviour.
- Then, if we draw a budget line for B, then the pt where that budget line intersects his offer curve represents B's optimal consumption.
- Thus, if A wants to choose prices to offer to B that makes A as well off as possible, she should find that pt on B's offer curve where A has the highest utility → that is pt. X in Fig 5

Fig 5



- The optimal choice is characterised by a tangency condition → A's IC will be tangent to B's offer curve.

→ If B's offer curve cut A's IC, there would be some pt on B's offer curve that A preferred (check diagram) — so we couldn't be at the optimal pt. for A.

→ • Once, we have identified this pt. X in Fig 5 — we just draw a budget line to that pt from the endowment

→ At the prices that generate this budget line, B will choose X and A will be as well off as possible.

Q - Is this allocation (X) Pareto-efficient?

Ans: NO → A's IC is not tangent to the budget line at X and ∴ A's IC is not tangent to B's IC. (rather it is tangent to B's offer curve).

- Hence, X → the monopoly allocation is Pareto-inefficient (shaded region — ~~scope for further~~ intersection of preferred regions exist)

In fact it is Pareto-inefficient in exactly the same way as described in the discussion of monopoly. < where deadweight loss occurs and gains from trade is not fully exhausted. → check?? >.

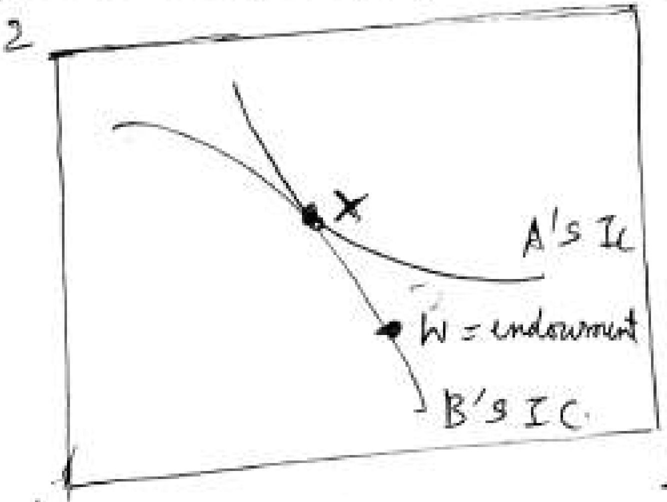
→ At the margin A would like to sell more at the expensive price, but she can only do so by lowering the price at which she sells and this will lower her income received from all her intramarginal sales

[⊛ I think that, this is only a part of the story. HAS TO BE CLARIFIED FURTHER

⊛ Later we found that a perfectly discriminating monopolist would end up producing an efficient level of output

Q. → How do we fit a case like this in the Edgeworth Box?

Ans: Fig-6.1



Let us start at the initial endowment W, and imagine A selling each unit of 1 to B at a different price — the price at which B is just indifferent bet/n buying or not buying that unit of the good.

• Thus, after A sells the 1st unit, B will remain on the same IC through W.

• Then, A sells the 2nd unit of 1 to B for the maximum price she is willing to pay. ⇒ that the allocation moves further to the left (→ because A is selling and B is buying) BUT, remains on the same IC through W.

• A continues to sell units to B in this manner thereby moving up B's IC to find A's most preferred pt — denoted by X in Fig 6.

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(4) It is clear that such a pt (X) is Pareto-efficient.
→ A will be as well off as possible given B's IC.
→ At such a pt, A has managed to extract all consumer's surplus from B.
⇒ B is no better off than she was at her endowment.

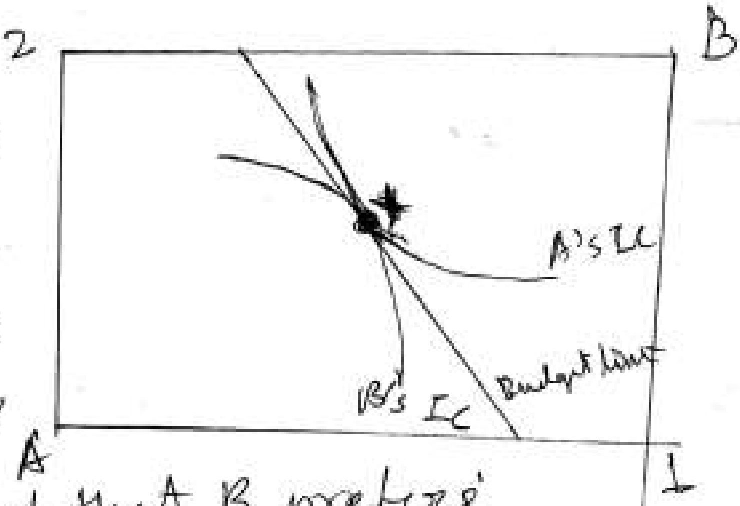
▶ **Efficiency and Equilibrium:**

- The FWT says that the eqm in a set of competitive mkts is Pareto-efficient.

Q. What about the <u>other way around</u>?
↳ Given a Pareto-efficient allocation, can we find prices such that it is a mkt eqm?

Ans: **YES** → under certain conditions.

Fig-7:



- Let us pick a Pareto efficient allocation.
- Then we know that the set of allocations that A prefers to her current allocation is disjoint from the set that B prefers
⇒ the two ICs are tangent at the Pareto-efficient allocation

Now, let us draw in the st. line that is their common tangent as in Fig-7.

• Suppose that the st. line represents the agents' budget sets; then if each agent chooses the best bundle on her budget set, the resulting equm will be the original Pareto efficient allocation.

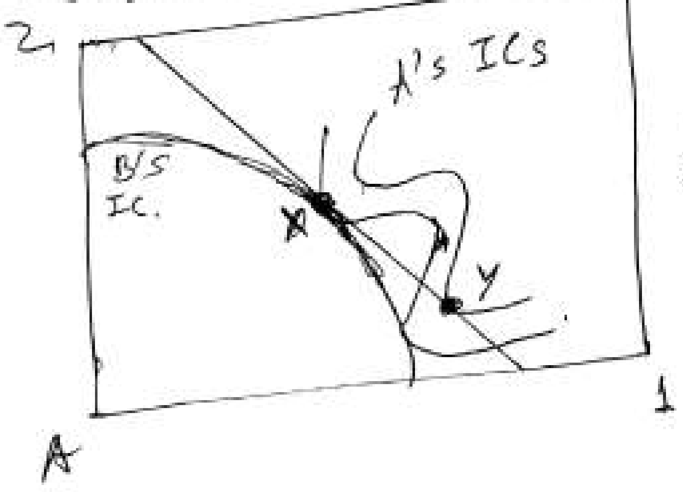
Thus, the fact that the original allocation is efficient automatically determines the equm prices.

→ The endowments can be any bundles that give rise to the appropriate budget set — that is, bundles that lie somewhere on the constructed budget line.

Q. Can the construction of such a budget line always be carried out?

Ans: NO

Fig: 8. (example)



Here the pt. X is Pareto efficient, BUT, there are no prices at which A and B will want to consume at pt X

→ The most obvious candidate is drawn in the diagram (?), but the optimal dde of A & B don't coincide for that budget: A wants to dd the bundle Y, but B wants the bundle X — dd ≠ ss at these prices.

→ The difference b/w Fig 7 and Fig 8 is that the preferences in the previous one is convex while the ones in the later are not.

If the preferences of both agents are convex, then the common tangent will not intersect either IC more than once, and everything will work out fine.

→ This observation, gives us the Second Theorem of Welfare Economics :-
if all agents have convex preferences, then there will always be a set of prices, such that each Pareto efficient allocation is a market equm for an appropriate assignment of endowments.

The proof is essentially the geometric argument given above.

Implications of the Welfare Theorems.

The two theorems of welfare economics are among the most fundamental results in economics. We have demonstrated the theorems only in the simple Edgeworth box case, but they are true for much more complex models with arbitrary numbers of consumers and goods. The welfare theorems have profound implications for the design of ways to allocate resources

FWT → any competitive equm. is Pareto efficient

Prepare short note from the book.