

MATHEMATICA

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DELTA Pg No.

Date / /

COMPUTER ALGEBRA SYSTEM (CAS)

A Computer Algebra System is a program with which you can perform calculations, evaluate functions, create graphics & develop your own programs. The key feature of computer algebra systems is the ability to manipulate expressions symbolically.

Mathematica is a popular computer algebra system. It was created by Stephen Wolfram is probably the world's most recognized CAS. One notable feature of Mathematica is its use of the 'notebook'. Mathematica notebooks allow a user to combine written text with calculations in one integrated document.

Mathematica as a calculator

To use Mathematica as a calculator, type the expression you wish to evaluate & press SHIFT + ENTER. (The special ENTER key on the lower-right corner of the keypad of an extended keyboard also works)

Some Mathematica funcs.

$\sqrt{5}$

e^x

$\ln 10$

$\log_{10} 5$

$\sin x$

Sqrt [5]

Exp [x]

Log [10]

Log [10, 5]

Sin [x]

Sign.

$\cos x$	$\text{Cos}[x]$
$\tan x$	$\text{Tan}[x]$
$\sum_{i=1}^n a_i$	$\text{Sum}[a[i], \{i, 1, n\}]$
$\prod_{i=1}^5 a_i$	$\text{Product}[a[i], \{i, 1, 5\}]$
$10 \bmod 3$	$\text{Mod}[10, 3]$

Examples

Write commands to express the follr

1. $7^{22} \bmod 23$

In [1]: $\text{Mod}[7^{22}, 23]$

2. $\log_{10}(5.65)$

In [2]: $\text{Log}[10, 5.65]$

3. $\sum_{i=1}^{n-1} \left(\frac{1+2i}{n}\right)^2$

In [3]: $\text{Sum}\left[\left(\frac{1+2*i}{n}\right)^2, \{i, 1, n-1\}\right]$

Matrix Operations:-

Let us define two matrices :-

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, N = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 5 & 6 \\ 0 & 2 & 0 \end{bmatrix}$$

These are entered as follows :-

$$\text{In}[4] := m = \left\{ \{1, 2, 3\}, \{4, 5, 6\}, \{7, 8, 9\} \right\}$$

$$\text{In}[5] := n = \left\{ \{0, 1, 0\}, \{2, 5, 6\}, \{0, 2, 0\} \right\}$$

Operations on matrices :-

$$\text{In}[6] := m + n$$

$$\text{Out}[6] := \left\{ \{1, 3, 3\}, \{6, 10, 12\}, \{7, 10, 9\} \right\}$$

$$\text{In}[7] := m \cdot n$$

$$\text{Out}[7] := \left\{ \{4, 17, 12\}, \{10, 41, 30\}, \{16, 65, 48\} \right\}$$

FUNCTIONS IN MATHEMATICA :

Mathematica contains many built-in functions. Each function name begins with a capital letter:-

1. $\text{Sin}[x]$, $\text{Cos}[x]$, $\text{Tan}[x]$

2. Binomial coeff. $\binom{7}{2} = \text{Binomial}[7, 2] = 7C_2 = 21.$

3. $\text{FactorInteger}[x] =$ prime factorization of $x.$

4. $\text{Prime}[n] =$ gives the n th prime number.

Ques (i) Write command to print first 10 prime numbers.

soln) $\text{In}[8] := \text{Table}[\text{Prime}[n], \{n, 1, 10\}]$

$$\text{Out}[8] := \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$$

Differentiation & Integration :-

* We define a function $f(x) = x^3 + \sin x$

In [9] :- $f[x] := x^3 + \sin[x]$

In [10] :- $D[f(x), x]$ { Differentiate }

Out [10] :- $3x^2 + \cos[x]$

In [11] :- $\text{Integrate}[f(x), x]$ { Integrate }

Out [11] :- $\frac{x^4}{4} - \cos[x]$

Que 2 Write command to evaluate

$$\int_{1/4}^{1/2} \frac{1}{x^2} dx$$

Soln. 2 In [12] :- $\text{Integrate}[1/x^2, \{x, 1/4, 1/2\}]$

Graphs

Mathematica offers many graphing options. One can use $\text{Plot}[]$ command

1. In [12] :- $\text{Plot}[\sin[x], \{x, 0, 2\pi\}]$

{ It plots the curve $y = \sin x$ in the range $0 \leq x \leq 2\pi$ }

2. One can plot more than one curve in a single curve:-

In [13] :- ~~Plot~~ $f[x] := 4.5x^2 + 1.2$

In [14] :- $\text{Plot}[\{f[x], \sin[x], \tan[x]\}, \{x, 0, 4\pi\}]$

Plots three funcs in a single plot.

Matrix Operations:

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For the matrix $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 4 & -1 \\ 2 & 5 & 3 \end{bmatrix}$

Write commands for the foll:-

(i) Diagonalization:

In[1] :- $m = \{ \{1, 3, 2\}, \{2, 4, -1\}, \{2, 5, 3\} \}$

Out[1] :- $\{ \{1, 3, 2\}, \{2, 4, -1\}, \{2, 5, 3\} \}$

In[2] = Eigenvalues[m]

Out[2] = $\{ \frac{1}{2}(5 + \sqrt{29}), 3, \frac{1}{2}(5 - \sqrt{29}) \}$

In[3] = {a, b, c} = Eigenvalues[m]

Out[3] = $\{ \frac{1}{2}(5 + \sqrt{29}), 3, \frac{1}{2}(5 - \sqrt{29}) \}$

In[4] = ans = $\{ \{a, 0, 0\}, \{0, b, 0\}, \{0, 0, c\} \}$

In[5] = MatrixForm[ans]

Out[5] = $\begin{bmatrix} \frac{1}{2}(5 + \sqrt{29}) & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & \frac{1}{2}(5 - \sqrt{29}) \end{bmatrix}$

(ii) Find inverse :

In[1]: Inverse[m]

Out[1]: $\left\{ \left\{ -\frac{17}{3}, -\frac{1}{3}, \frac{11}{3} \right\}, \left\{ \frac{8}{3}, \frac{1}{3}, -\frac{5}{3} \right\}, \left\{ -\frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right\} \right\}$

How to Solve an equation

(i) Equation in one variable :-

DO-19

Ques write command to evaluate $2x^2 + x = 1$

Solve: In[1]: Solve $[2 * x^2 + x == 1, x]$

Out[1]: $\left\{ \{x \rightarrow -1\}, \{x \rightarrow \frac{1}{2}\} \right\}$

(ii) Simultaneous eqns in two variables :-

In[1]: Solve $\left[\left\{ x - y == 0, x^2 + y^2 == 1, \{x, y\} \right\} \right]$

Out[1]: $\left\{ \left\{ x \rightarrow -\frac{1}{\sqrt{2}}, y \rightarrow -\frac{1}{\sqrt{2}} \right\}, \left\{ x \rightarrow \frac{1}{\sqrt{2}}, y \rightarrow \frac{1}{\sqrt{2}} \right\} \right\}$

DO-2019

Write a program to find the gcd of two integers a, b using Euclidean algorithm & hence find gcd of 120 & 75.

7

Sol.

GCDprog [a, b] :-

Module [[q, r, DebugFlag = True],

r = Mod [a, b]; (remainder)

while [r != 0,

q = Floor [(a/b)]; (quotient)

a = b;

b = r;

GCDprog [a, b];

]; (End of while Loop)

Return [b]

] (End of Module)