

## **ARSD College, University of Delhi**

Lesson Plan

Course Name : B.Sc. (H) Mathematics								
Semester	Course Code	Course Title	Lecture (L)	Tutorial (T)	Practical (P)	Credit (C)		
VI	BMATH614	Ring Theory and Linear Algebra-II	5	1				
Teacher/Instructor(s)		Mr. AGAM DWIVEDI						
Session		2022-23						

**Course Objective:** This course introduces the basic concepts of ring of polynomials and irreducibility tests for polynomials over ring of integers, used in finite fields with applications in Cryptography. This course emphasizes the application of techniques using the adjoint of a linear operator and their properties to least squares approximation and minimal solutions to systems of linear equations.

Course Learning Outcomes: On completion of this course, the student will be able to:

i) Appreciate the significance of unique factorization in rings and integral domains.

ii) Compute with the characteristic polynomial, eigenvalues, eigenvectors, and eigenspaces, as well as the geometric and the algebraic multiplicities of an eigenvalue and apply the basic diagonalization result.

iii) Compute inner products and determine orthogonality on vector spaces, including Gram-Schmidt orthogonalization to obtain orthonormal basis.

Unit No.	Learning Objective	Lecture No.	Topics to be covered	
1.	Polynomial rings	1-5	Polynomial rings over commutative rings, Division algorithm and consequences, Principal ideal domains.	
2.	FactorizationofpolynomialsandReducibility	6-15	Factorization of polynomials, Reducibility tests, Irreducibility tests, Eisenstein's criterion, Unique factorization in Z[x].	
3.	Divisibility in integral domains	16-25	Divisibility in integral domains, Irreducibles, Primes, Unique factorization domains, Euclidean domains.	
4.	Dual spaces	26-30	Dual spaces, Double dual, Dual basis, Transpose of a linear transformation and its matrix in the dual basis, Annihilators.	
5.	Eigenvalues and Eigenvectors	31-40	Eigenvalues, Eigenvectors, Eigenspaces and characteristic polynomial of a linear operator; Diagonalizability, Invariant subspaces and Cayley-Hamilton theorem; The minimal polynomial for a linear operator	
6.	Inner product space	41-45	Inner product spaces and norms	
7.	orthogonalization	46-55	Orthonormal basis, Gram-Schmidt orthogonalizatio process, Orthogonal complements, Bessel's inequality.	
8.	adjoint of a linear operator 56-		The adjoint of a linear operator and its properties, Leas squares approximation, Minimal solutions to systems of linear equations	

## Lesson Plan:

9.	Normal, Self-adjoint, unitary and orthogonal operators	61-70	Normal, Self-adjoint, unitary and orthogonal operators and their properties

## **Evaluation Scheme:**

No.	Component	Duration	Marks
1.	Internal Assessment		
	Quiz		
	Class Test		25
	Attendance		
	Assignment		
2.	End Semester Examination	3 hr	75

Details of the Course							
Unit	Contents						
1	Polynomial rings over commutative rings, Division algorithm and consequences, Principal ideal domains.			05			
2	Factorization of polynomials, Reducibility tests, Irreducibility tests, Eisenstein's criterion. Unique factorization in Z[x].						
3	Divisibility in integral domains, Irreducibles, Primes, Unique factorization domains, Euclidean domains.						
4	Dual spaces, Double dual, Dual basis, Transpose of a linear transformation and its matrix in the dual basis. Annihilators.						
5.	Eigenvalues, Eigenvectors, Eigenspaces and characteristic polynomial of a linear operator; Diagonalizability, Invariant subspaces and Cayley-Hamilton theorem; The minimal polynomial for a linear operator.						
6.	Inner product spaces	and norms.		05			
7.	Orthonormal basis, Gram-Schmidt orthogonalization process, Orthogonal complements, Bessel's inequality.						
8.	The adjoint of a linear operator and its properties, Least squares approximation, Minimal solutions to systems of linear equations						
9.	Normal, Self-adjoint,	unitary and orthogonal operators and their properties.		10			
	Total						
Suggestee	d Books:						
Sl. No.	No. Name of Authors/Books/Publishers Year Publicationt			of ion/Repri			
1	Friedberg, Stephen H., Insel, Arnold J., & Spence, Lawrence E. (2003).2003Linear2003Algebra (4th ed.). Prentice-Hall of India Pvt. Ltd. New Delhi.						
2	Gallian, Joseph. A. (2013). Contemporary Abstract Algebra (8th ed.).2015Cengage Learning India Private Limited. Delhi. Fourth impression, 2015.2015						
3	Herstein, I. N. (2006). Topics in Algebra (2nd ed.). Wiley Student Edition. 2006 India						
4.	Hoffman, Kenneth, & Kunze, Ray Alden (1978). Linear Algebra (2nd ed.).Prentice-Hall of India Pvt. Limited. Delhi. Pearson Education India Reprint,2015.						
Mode of Evaluation: Internal Assessment / End Semester Exam							

Mr. AGAM DWIVEDI Assistant Professor Department of Mathematics